

## Introduction to Commutative Algebra and algebraic Geometry Presence Exercise to Sheet 5

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### Exercise 1.

Let  $f = x^2y^3 - z + 3x^2z + xy - 10y^2z + xz^2, g = x^4z^2 + 7y^3z + z^2 - 3xy + 5 \in K[x, y, z]$ . Write  $f, g$  in the

- lexicographic order with  $z > y > x$ .
- degree reverse lexicographic order with  $y > z > x$ .
- the weighted degree reverse lexicographic order given by  $w = (2, -1, 1)$  for which the first coordinate corresponds to  $x$ , the second to  $y$  and the third to  $z$  (i.e.  $x_1 = x, x_2 = y, x_3 = z$ .)

### Exercise 2.

Matrices  $A \in \text{GL}(n, \mathbb{R})$  with real entries can be used to obtain a monomial ordering on  $\text{Mon}_n$  by setting

$$x^\alpha >_A x^\beta \Leftrightarrow A\alpha > A\beta$$

where  $>$  on the right hand side is the lexicographical ordering on  $\mathbb{R}^n$ .

Find the matrices that induce the lexicographic and the degree reverse lexicographic ordering on  $\text{Mon}_n$  with  $X_1 > \dots > X_n$ .

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