Eberhard-Karls-Universität Tübingen
Prof. Hannah Markwig
Alheydis Geiger

Wintersemester 2020/2021
16. December 2020

Introduction to Commutative Algebra and algebraic Geometry
Presence Exercise to Sheet 6

Exercise 1.
For $R = \mathbb{Q}[x, y, z]$ compute spoly$(xy - y, 2x^2 + yz)$ and spoly$(xy - y, -y^3 - 2y)$ with respect to $>_{lp}$.

Exercise 2.
A polynomial $f = \sum_{\alpha \in \mathbb{N}^n} a_\alpha \underline{x}^\alpha \in K[x_1, \ldots, x_n]$ is called *homogeneous* if for all α with $a_\alpha \neq 0$ the absolute value $|\alpha|$ is constant.
An ideal $I \subset K[x_1, \ldots, x_n]$ is called *homogeneous* if it is generated by a set of homogeneous polynomials.

Let I be a homogeneous ideal in $K[\underline{x}]$. Show that the degree reverse lexicographic ordering $>_{dp}$ satisfies the property:

$$L(I + \langle x_n^d \rangle) = L(I) + \langle x_n^d \rangle \text{ for any } d \geq 1.$$

Hint: Show that if I is homogeneous, we only have to consider homogeneous polynomials, e.g. $L(I) = \langle LM(f) | f \in I, f \text{ homogeneous} \rangle$.
You are allowed to use that the sum of homogeneous ideals is again a homogeneous ideal.