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Introduction to Commutative Algebra and algebraic Geometry Presence Exercise to Sheet 12

Exercise 1.
Let K be an algebraically closed field. Let X be an affine variety.
Prove: There is a canonical bijection $\mathrm{Mor}(X, K^n) \to O(X)^n$, $\varphi \mapsto (\varphi_1, \dots, \varphi_n)$.

Solution: We know from exercise sheet 10, that $\mathcal{O}(X) = \mathrm{Mor}(X, K)$. It follows that every element $(\phi_1, \dots, \phi_n) \in \mathcal{O}(X)^n$ gives rise to a morphism $X \to K^n$ via $x \mapsto (\phi_1(x), \dots, \phi_n(x))$.
In the other direction every morphism $X \to K^n$ gives rise to a morphism $X \to K$ (which is an element in $\mathcal{O}(X)$) by combining it with a coordinate projection $\pi_i : K^n \to K, (a_1, ..., a_n) \to a_i$.
Thus, we obtain an element in $\mathcal{O}(X)^n$ from $\phi \in \mathrm{Mor}(X, K^n)$ via $\phi \mapsto (\pi_1 \circ \phi, \dots, \pi_n \circ \phi)$.
These two operations are inverse to each other thus providing the bijection.

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