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Wintersemester 2020/2021
2. December 2020

Introduction to Commutative Algebra and algebraic Geometry face-to-face Exercise to Sheet 4

Exercise 1.
Determine the irreducible components of $V(X^2 - YZ, XZ - X) \subset \mathbb{C}^3$.

Solution:

$$XZ - X = 0$$
$$\Rightarrow X = 0 \text{ or } Z = 1$$

Since additionally $X^2 - YZ = 0$, we have $YZ = 0$ for $X = 0$, and $X^2 - Y = 0$ for $Z = 1$.
So

$$V(X^2 - YZ, XZ - Z) = V(X, Y) \cup V(X, Z) \cup V(Z - 1, X^2 - Y).$$

$I(V(X,Y)) = \langle X, Y\rangle$ and $I(V(X,Z)) = \langle X, Z\rangle$ are prime. Also $I(V(Z-1, X^2-Y)) = \langle Z-1, X^2-Y\rangle$ is prime, since $K[X,Y,Z]/\langle Z-1, X^2-Y\rangle \cong K[X,Y]/\langle X^2-Y\rangle$ is an integer domain (since $X^2 - Y$ is irreducible and thus prime). So we have found the decomposition of the irreducible components.