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Introduction to Commutative Algebra and algebraic Geometry
Presence Exercise to Sheet 9

Exercise 1.
Let K be an algebraically closed field. Determine the algebra of regular functions $\mathcal{O}(U_i)$ for the following open sets U_i:

- $U_1 = K$,
- $U_2 = K \setminus \{0, 1\}$,
- $U_3 = (X \setminus V(f)) \subset X$, where $X = V(x \cdot y \cdot z, x^2 - xy^3) \subset K^3$ and $f = x^2 + y^2 + z^2 - 1 \in K[x, y, z]$.

Solution:

- $U_1 = K$,: $\mathcal{O}(K) = K[X]$, since we know that the coordinate ring to K is $K[X]$ and by choosing $X = K$ and $g = 1$ in 3.2.9 we obtain the claim.
- $U_2 = K \setminus \{0, 1\}$: We apply 3.2.9 for $X = K$ and $g = X(X - 1)$. We obtain: $\mathcal{O}(K \setminus \{0, 1\}) = K[X]_{X(X-1)}$.
- $U_3 = (X \setminus V(f)) \subset X$, $X = V(x \cdot y \cdot z, x^2 - xy^3) \subset K^3$ and $f = x^2 + y^2 + z^2 - 1 \in K[x, y, z]$. We know $K[X] = K[x, y, z]/I(X)$ and $I(X) = \sqrt{\langle x \cdot y \cdot z, x^2 - xy^3 \rangle}(= \langle x \cdot z, x - x^2y^3 \rangle)$.
  Now $X \setminus V(f)$ is the same as $X \cap D(g)$ where $g = \overline{f} \in K[x, y, z]/I(X)$. By 3.2.9 we know that $\mathcal{O}(U_3) = K[X]_g = (K[x, y, z]/I(X))_{\overline{x^2+y^2+z^2-1}}$. □