

Exercise Sheet 10

Introduction to Commutative Algebra and Algebraic Geometry

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Exercise 1.

Exercise 1. Let K be an algebraically closed field and let X be an affine variety. Let $f : X \rightarrow K$ be a map. Prove that the following statements are equivalent:

- (i) $f \in \mathcal{O}_X(X)$.
- (ii) $f : X \rightarrow K$ is a morphism.

Exercise 2.

Let X be an affine variety and \mathcal{F} its sheaf of regular functions. Let $U \subset X$ be an open set. Let $s \in \mathcal{F}(U)$ be an element with $s_x = 0 \in \mathcal{F}_x$ for all $x \in U$. Show: $s = 0$.

Exercise 3.

Let K be an algebraically closed field with $\text{char}(K) = 0$. Consider the map from the affine line \mathbb{A}_K^1 to the curve

$$C = V(y^2 - x^3)$$

given by

$$\phi : \mathbb{A}_K^1 \rightarrow C, \quad t \mapsto (t^2, t^3).$$

Prove ϕ is a morphism and a homeomorphism (i.e. bijective, continuous and open).