Exercise Sheet 9
Introduction to Commutative Algebra and Algebraic Geometry

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Exercise 1.

Let K be an algebraically closed field,

$$A := K[T_1, T_2]/\langle T_1 \cdot T_2 \rangle$$

and $f \in A$ the equivalence class of T_1. Prove:

$$A_f \cong K[T]_T.$$

Exercise 2.

Let R be a commutative ring with one and let $S \subset R$ be multiplicatively closed. Show: If R is noetherian, then $S^{-1}R$ is noetherian.

Exercise 3.

Let K be an algebraically closed field. Determine the algebra of regular functions $\mathcal{O}(U_i)$ for the following open sets U_i:

- $U_1 = K \setminus \{0\} \subset K$,
- $U_2 = K^\times \times K \subset K^2$,
- $U_3 = K^\times \times K^\times \subset K^2$.

Submission: Work in groups of up to three students. Submit your solutions either by uploading them to URM or by placing them in the mailbox of Parisa Ebrahimian or Veronika Körber (Room A16, C-Building) by Tuesday, 16 Dec 2025