Universität Tübingen, SS 2018
Topics in Riemannian Geometry
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Problem Set 3

Problem 3.1 – Isometries III

Consider the hyperbolic plane $H = \{x + iy \in \mathbb{C} \,|\, y > 0\}$ equipped with the metric $g = (dx^2 + dy^2)/y^2$. Determine all geodesics of H.

Problem 3.2 – Extendible manifolds

Let M be a Riemannian manifold. We say that M is geodesically complete if any geodesic $\gamma(t)$ starting from p is defined for all $t \in \mathbb{R}$. We say that M is extendible if there exists a connected Riemannian manifold N such that M is isometric to a proper open subset of N. Show that if M is geodesically complete, then it is non-extendible.

Problem 3.3 – Convergence to infinity

Let M be a Riemannian manifold. A curve $\gamma : [0, b) \to M$, $0 < b \leq \infty$, is said to converge to infinity if for all compact sets $K \subset M$, there is a time $T \in [0, b)$ such that $\gamma(t) \notin K$ for all $t > T$. Show that M is geodesically complete if and only if every piecewise smooth curve that converges to infinity has infinite length, where the length of γ is given by

$$\ell(\gamma) = \sup_{a<b} \ell(\gamma|_{[0,a]}).$$