Universität Tübingen, SS 2018
Topics in Riemannian Geometry
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**Problem Set 6**

### Problem 6.1 – Covering maps

Let $M$ and $\tilde{M}$ be a manifolds, and let $\pi : \tilde{M} \to M$ be a smooth covering map, i.e., $\pi$ is a smooth, surjective map, and each point in $M$ has a neighbourhood $U$ such that each component of $\pi^{-1}(U)$ is mapped diffeomorphically onto $U$ by $\pi$. We say that a smooth map $\phi : \tilde{M} \to \tilde{M}$ is a covering transformation if $\pi \circ \phi = \pi$.

a) Show that if $g$ is a metric on $M$, then $\pi^* g$ is a metric on $\tilde{M}$ that is invariant under all covering transformations.

b) Suppose that $\pi$ is also normal, i.e., that the group of covering transformations acts transitively on each fibre of $\pi$. Show that if $h$ is a metric on $\tilde{M}$ that is invariant under all covering transformations, then then there exists a unique metric $g$ on $M$ such that $h = \pi^* g$.

### Problem 6.2 – Covering Spaces and Completeness

Let $\pi : M \to N$ be a smooth covering map that is also a local isometry. Show that $M$ is complete if and only if $N$ is complete.