Summer School “Paradoxes in Quantum Physics”

September 2-6, 2019

The John Bell Institute

with lectures by Jean Bricmont, Matthias Lienert, Roderich Tumulka, and Lev Vaidman

JOHN BELL INSTITUTE FOR THE FOUNDATIONS of PHYSICS

Founded by philosopher Tim Maudlin (New York City) in 2018 for the advancement of knowledge on foundational questions of physics.

Our instructors this week

Jean Bricmont

- Professor emeritus at Université Catholique in Louvain (Belgium)
- Extensive work on mathematical and theoretical physics, particularly in statistical mechanics and statistical quantum mechanics (renormalization group, Fourier's law . . .)
- Books:
- *Making sense of quantum mechanics* (2016)
- *Quantum Sense and Nonsense* (2017) (for the public)
- Also known for his book *Fashionable Nonsense* (1997) with Alan Sokal, in which he criticized postmodern philosophers for misuse of science.

Our instructors this week

Lev Vaidman

- Professor of theoretical physics at Tel Aviv University (Israel)
- Extensive work in the foundations of quantum mechanics, particularly on the many-worlds interpretation of quantum mechanics.
- He is known particularly for work on quantum teleportation, weak values, the Elitzur-Vaidman bomb-tester, and numerous paradoxes in quantum physics.

Our instructors this week

Matthias Lienert

- Postdoctoral researcher in the math department at Eberhard Karls University Tübingen (Germany)
- Ph.D. from Ludwig Maximilians University Munich (Germany), worked at Rutgers University (USA)
- Works in mathematical physics, particularly covariant formulations of relativistic quantum mechanics (multi-time wave functions)

Our instructors this week

Roderich Tumulka

- Professor at Eberhard Karls University Tübingen (Germany)
- Taught 9 years at Rutgers University (USA)
- I work in mathematical physics, particularly on foundational questions of quantum mechanics, relativity, and quantum statistical mechanics.

- We will have a question-and-answer period at the end of each lecture.
- Students get priority for asking questions.
- Please feel free to ask questions any time, also in the middle of lectures.
- Also, feel free to approach the instructors with questions during breaks and meals.
- In the exercise sessions on Tuesday and Thursday, you will solve problems to recapitulate and deepen the material of the lectures. Instructors are available to answer your questions.
- Please sign up for the group dinners if you want to join.

Introduction

Roderich Tumulka

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

September 2, 2019

Summer school "Paradoxes in Quantum Physics"
John Bell Institute, Croatia

Paradoxes

- Epimenides the Cretan said, “All Cretans lie.”

Paradoxes

- "This sentence is false."

Paradoxes

- "This sentence is false."
- A logical paradox
- Common resolution: The statement doesn't have a truth value. Formalized logic excludes it from the class of meaningful statements.

- General pattern: A paradox provides a plausible argument for a statement A and another plausible argument for the negation of A.
- (Related to a *dilemma*: Of two statements, A and B, one must be right, but there are plausible arguments against both.)
- If S is a meaningful statement (not a logical paradox), one of the arguments must be wrong.

Some paradoxes are basically a proof by contradiction. E.g.,

Russell's paradox in set theory

Let M be the set of all sets that do not contain themselves, $M = \{S : S \notin S\}$.

Statement A: $M \in M$. Proof: If $M \notin M$, then $M \in M$, contradicting the assumption, so we have to drop the assumption. $\square$

Proof of non-A: If $M \in M$, then $M \notin M$, contradicting the assumption, so we have to drop the assumption. $\square$

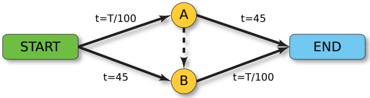
Resolution: Both arguments made use of the hidden assumption that for every assertion $p(S)$ that a set S may or may not satisfy, there exists a set of all sets S satisfying p, $\exists\{S : p(S)\}$. But this assumption is refuted by the contradiction.

Upshot: $\exists p \nexists \{S : p(S)\}$

Some paradoxes are just surprising statements: Get used to the fact!

Braess's paradox

In a road network, 4000 drivers want to go from Start to End. $T =$ number of drivers on this road. If every driver minimizes their time, then $T(\text{Start-A}) = T(\text{B-End})$, and the time from Start to End is 65.
Now a fast A-B road is built. It takes about 0 minutes. If every driver minimizes their time, the time from Start to End becomes 85.



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Banach-Tarski paradox

The unit sphere in $\mathbb{R}^3$ can be partitioned into 10 subsets $A_1, \ldots, A_{10}$, and there are 10 rotations $R_1, \ldots, R_{10} \in SO(3)$, such that $R_1 A_1, \ldots, R_5 A_5$ form a partition of the unit sphere and $R_6 A_6, \ldots, R_{10} A_{10}$ form a partition of the unit sphere.

From one sphere make two! This would be impossible if $A_1, \ldots, A_{10}$ were measurable, but they are not. (Controversial b/c uses axiom of choice)

- Paradoxes also come up in quantum mechanics.
- More than that, paradoxes play a crucial role in the Copenhagen interpretation of quantum mechanics, which ges back to Niels Bohr. That is because Bohr's idea of complementarity has something to do with paradoxes. Let me explain.

Einstein (1949):

"Despite much effort which I have expended on it, I have been unable to achieve a sharp formulation of Bohr's principle of complementarity."

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Bell commented (1986):

"What hope then for the rest of us?"

How Bohr defined complementarity:

"Any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena."

How I understand Bohr's idea:

In order to compute a quantity of interest (e.g., the wave length of light scattered off an electron), we use both Theory A (e.g., classical theory of billiard balls) and Theory B (e.g., classical theory of waves) although A and B contradict each other. It is impossible to find one Theory C that replaces both A and B and explains the entire physical process. Instead, we should leave the conflict between A and B unresolved and accept the idea that reality is paradoxical.

How Bell explained complementarity:

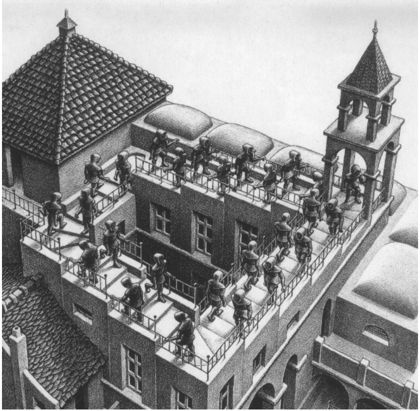
"It seems to me that Bohr used this word with the reverse of its usual meaning. Consider for example the elephant. From the front she is head, trunk and two legs. From the back she is bottom, tail, and two legs. From the sides she is otherwise, and from the top and bottom different again. These various views are complementary in the usual sense of the word. They supplement one another, they are consistent with one another, and they are all entailed by the unifying concept 'elephant.' It is my impression that to suppose Bohr used the word 'complementary' in this ordinary way would have been regarded by him as missing his point and trivializing his thought. He seems to insist rather that we must use in our analysis elements which *contradict* one another, which do not add up to, or derive from, a whole. By 'complementarity' he meant, it seems to me, the reverse: contradictoriness."

- To reiterate the last point in my own words:
- According to key elements of the Copenhagen view,
  - reality itself is contradictory.
  - That is why there is no Theory C, no single picture that completely describes reality.
  - At the same time, we can never observe a contradiction in experiment (e.g., because we can only observe one of two complementary observables).
  - And since we cannot observe contradictions, the contradictions are somehow not a problem. (My understanding of Bohr)

According to Copenhagen, it is like in the Charles Addams cartoon:

We never see the paradoxical thing happen. But we see traces showing that it must have happened.

- According to Copenhagen, some paradoxes just cannot be resolved.
- This week, we would like to find out whether that is so.
- We want to get to the bottom of the paradoxes in quantum physics.
- For this, we need to scrutinize theories claiming to be such a "Theory C," claiming to resolve the paradoxes of quantum physics, claiming to be able to provide a single coherent picture of how the quantum world works.
- The leading theories of this kind are: Bohmian mechanics, many-worlds, and the Ghirardi-Rimini-Weber collapse theory. We will scrutinize how these theories work and what they can or cannot achieve.



- Would you believe that something paradoxical can be real?
- Would you believe that M.C. Escher's paradoxical stairwell could exist in reality?
- Michael Lacanilao, who is visiting the John Bell Institute this week, tried whether he could convince people by means of a YouTube video that it could: `http://www.youtube.com/watch?v=iBY4HaAngaA`

All views about QM agree about the rules for making empirical predictions:

- Unitary evolution: The wave function φ of an isolated system evolves according to the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\tfrac{\hbar^2}{2m}\sum_{j=1}^{N}\nabla^2_{\boldsymbol{q}_j}\psi + V\psi\,.$$

- Born's rule: When an observer makes an "ideal quantum measurement" of the observable $\mathscr{A}$ associated with the self-adjoint operator A with spectral decomposition $A=\sum_\alpha \alpha P_\alpha$ on a system with wave function ψ, the outcome is the eigenvalue α with probability $\|P_\alpha\psi\|^2 = \langle\psi|P_\alpha\psi\rangle$.
- Collapse rule: After an ideal quantum measurement of $\mathscr{A}$ with outcome α, the wave function gets replaced by

$$\psi_{t+} = \frac{P_\alpha\psi_{t-}}{\|P_\alpha\psi_{t-}\|}\,.$$

Limitations to knowledge (1)

In any version of QM, you cannot measure the wave function.

- Example: Alice chooses a direction $\boldsymbol{a}$ in space and prepares a spin-$\frac{1}{2}$ particle with $\psi = |\text{spin up in } \boldsymbol{a}\rangle$.
- She hands it to Bob with the challenge to determine ψ (or $\boldsymbol{a}$).
- According to the rules of QM, Bob can do no better than perform a Stern-Gerlach experiment in a direction $\boldsymbol{b}$ of his choice and obtain 1 bit ("up" or "down").
- He can conclude whether $\boldsymbol{a}$ is more likely to lie in the hemisphere closer to $\boldsymbol{b}$ or closer to $-\boldsymbol{b}$, but cannot determine $\boldsymbol{a}$.
- (If the game is repeated and Alice always prepares the same ψ, Bob can determine ψ to desired accuracy. But not in a single run.)
- Nature knows in every single run what ψ is because Alice knows, and she can prove it. So,

Limitation to knowledge

Certain variables have well-defined values in the world (known to nature), although we cannot measure them, even with all future advances.

Limitations to knowledge (2)

Limitations to knowledge may seem to conflict with some principle of science, such as

> *"a statement is unscientific or even meaningless if it cannot be tested experimentally, an object is not real if it cannot be observed, and a variable is not well-defined if it cannot be measured."*

- But limitations to knowledge are a fact of quantum mechanics.
- Get used to them!
- The "principle" above is not a principle at all, it is wrong. It is exaggerated positivism.

Limitations to knowledge (3)

- are unknown in classical physics but common in quantum physics.
- Another example:
- Suppose $\{\varphi_1^a, \varphi_2^a\}$ and $\{\varphi_1^b, \varphi_2^b\}$ are two orthonormal bases of $\mathscr{H}$.
- Alice chooses either $i = a$ or $i = b$ and prepares an ensemble of particles, each in φ_1^i with prob $\frac{1}{2}$ and φ_2^i with prob $\frac{1}{2}$.
- The particles are handed over to Bob, who is asked whether $i = a$ or $i = b$.
- Claim: Bob can't find out empirically. The two ensembles are empirically indistinguishable.
- This is another limitation to knowledge: There is a fact whether in reality the ensemble is a or b. (Nature knows because Alice knows. She knows the state vector of every single particle, and she can prove it.)
- How to prove the claim?
- For every projection operator P, Bob's probability of the corresponding outcome is $\frac{1}{2}\langle\varphi_1^i|P|\varphi_1^i\rangle + \frac{1}{2}\langle\varphi_2^i|P|\varphi_2^i\rangle = \frac{1}{2}\,\mathrm{tr}\,P$.
- If Bob can do arbitrary experiments, not just a single ideal quantum measurement, need POVMs. (Later this week)

- General ensemble: prob distribution μ on unit sphere $\mathbb{S}$ in $\mathscr{H}$.
- Prob(outcome "+" when measuring P) $=$ $\displaystyle\int_{\mathbb{S}} \mu(d\psi)\, \langle\psi|P|\psi\rangle = \mathrm{tr}(P\rho)$
- with statistical density matrix $\rho = \displaystyle\int_{\mathbb{S}} \mu(d\psi)\, |\psi\rangle\langle\psi|$.
- Two ensembles μ, μ' are empirically indistinguishable iff (if and only if) they have the same density matrix, $\rho = \rho'$.
- Example: φ_1^i with prob $\frac{1}{2}$ and φ_2^i with prob $\frac{1}{2}$ leads to $\rho = \frac{1}{2} I$.
- Example: uniform distribution over $\mathbb{S}$ leads to $\rho = \frac{1}{d} I$ ($d = \dim \mathscr{H}$).
- An operator $\rho : \mathscr{H} \to \mathscr{H}$ can occur as a statistical density matrix if and only if
    - ρ is a positive operator (i.e., self-adjoint with spectrum in $[0, \infty)$)
    - and $\mathrm{tr}\, \rho = 1$.
- Such an operator ρ is called pure iff $\rho = |\chi\rangle\langle\chi|$ (1d projection) and mixed otherwise.

Reduced density matrix

- Consider composite quantum system $a \cup b$, $\psi \in \mathscr{H}_a \otimes \mathscr{H}_b$.
- Suppose Bob can make experiments only on a, $P = P_\alpha \otimes I_b$.
- Then $\text{Prob}(\text{outcome } \alpha) = \langle \psi | P_\alpha \otimes I_b | \psi \rangle = \text{tr}(P_\alpha \rho)$ with
- reduced density matrix $\rho = \text{tr}_b \, |\psi\rangle\langle\psi|$,
- where tr_b is the partial trace,
  $$\langle \varphi_i^a | \, \text{tr}_b \, S | \varphi_j^a \rangle = \sum_k \langle \varphi_i^a \otimes \varphi_k^b | S | \varphi_j^a \otimes \varphi_k^b \rangle .$$
- Note $\rho : \mathscr{H}_a \to \mathscr{H}_a$.
- An operator can occur as a reduced density matrix iff it is positive and has trace 1.
- Such an operator ρ is called pure iff $\rho = |\chi\rangle\langle\chi|$ (1d projection) and mixed otherwise, *even though* we are not talking about a mixture.

Thank you for your attention