Relativistic Quantum Mechanics

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Motivation

There are many issues in relativistic quantum physics. To name a few:

1. negative energies?
2. problem of adequate position operators, Malament's theorem
3. particle creation/annihilation
4. problem of consistent interactions (ultraviolet divergences)
5. ...

Here: focus on the question raised by Bell's theorem:

Question

Can the non-locality of QM be reconciled with relativity? If so, how? If not fully, to which extent?

Overview

To answer the question, one needs a precise version of QM (which is free of the measurement problem). We will discuss how the theories we already got to know can be combined with (special) relativity.

Outline:

1. Multi-time wave functions (concept of a relativistic wave fn.)
2. Relativistic version of the MWI
3. Relativistic version of GRWf
4. Hypersurface Bohm-Dirac model

Multi-time wave functions

Background

In a relativistic situation, the basic object of QM, the wave fn., should be a **Lorentz-covariant object.**

For a single particle, this is achieved by the **Dirac equation**

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0.$$

Under a **Lorentz transformation** Λ, ψ transforms as:

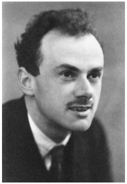
$$\psi(x) \to \psi'(x) = S[\Lambda]\psi(\Lambda^{-1}x)$$

where $S[\Lambda]$ is a 4×4 matrix belonging to the **spinor representation** of the Lorentz group ($SO(1,3)$).

However: for $N \geq 2$, $\psi(t, \mathbf{x}_1, ..., \mathbf{x}_N)$ is not a Lorentz covariant object. It is unclear how to transform a function of one time variable and many space variables.

Idea of a multi-time wave function

$$\psi(\mathbf{x}_1, ..., \mathbf{x}_N, t) = \phi(t_1, \mathbf{x}_1, ..., t_N, \mathbf{x}_N)\big|_{t_1=\cdots=t_N=t}$$



Dirac

Tomonaga

Schwinger

Basic object: multi-time wave function

$$\phi : \mathscr{S} \subset \underbrace{\mathbb{R}^4 \times \cdots \times \mathbb{R}^4}_{N} \rightarrow \mathcal{S},$$

$$(t_1, \mathbf{x}_1, ..., t_N, \mathbf{x}_N) \equiv (x_1, ..., x_N) \longmapsto \phi(x_1, ..., x_N).$$

Domain: space-like configurations

$$\mathscr{S} = \{(x_1, ..., x_N) \in \mathbb{R}^{4N} | \forall j \neq k : (x_k - x_j)^2 < 0\}$$

Multi-time wave equations

Idea (Dirac 1932): system of N wave eqs., one for each particle:

Multi-time Schrödinger eqs.

$$i\frac{\partial\phi}{\partial t_k} = H_k\phi, \quad k = 1, ..., N.$$

H_k: differential operators ('partial Hamiltonians')
Eqs. should be re-writable in a covariant form.

Solvable for all initial data $\phi(t_1, \mathbf{x}_1, ..., t_N, \mathbf{x}_N) = \phi_0(\mathbf{x}_1, ..., \mathbf{x}_N)$ if and only if

Consistency condition

$$[i\partial_{t_k} - H_k, i\partial_{t_j} - H_j] = 0 \quad \forall j \neq k.$$

Multi-time wave equations

Relation to usual Schrödinger eq.: Use chain rule and the multi-time eqs. to show:

$$i\partial_t \phi(t, \mathbf{x}_1, ..., t, \mathbf{x}_N) = \sum_{k=1}^{N} H_k \phi(t, \mathbf{x}_1, ..., t, \mathbf{x}_N)$$

That means, the single-time wf. $\psi(t, \mathbf{x}_1, ..., \mathbf{x}_N) = \phi(t, \mathbf{x}_1, ..., t, \mathbf{x}_N)$ satisfies the usual Schrödinger eq.

Example for multi-time eqs.: free multi-time Dirac eqs.

$$(i\gamma_k^\mu \partial_{x_k^\mu} - m_k)\phi(x_1, ..., x_N) = 0, \quad k = 1, ..., N.$$

$$\gamma_k^\mu = 1 \otimes \cdots \underbrace{\otimes \gamma^\mu \otimes}_{k-\text{th position}} \cdots \otimes 1$$

Probability currents

Free multi-time Dirac eqs. imply **continuity eqs.**

$$\partial_{x_k^{\mu_k}} \overline{\phi}(x_1,...,x_N)\gamma_1^{\mu_1}\cdots\gamma_k^{\mu_k}\cdots\gamma_N^{\mu_N}\phi(x_1,...,x_N) = 0, \quad k = 1,...,N.$$

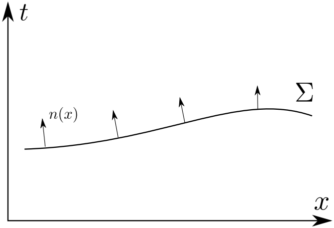
where $\overline{\phi} = \phi^\dagger\gamma_1^0\cdots\gamma_N^0$.

Tensor current: $j_\phi^{\mu_1\ldots\mu_N} = \overline{\phi}\gamma_1^{\mu_1}\cdots\gamma_N^{\mu_N}\phi$.

Probability density: Consider a space-like surface $\Sigma \subset \mathbb{R}^4$ with normal vector field $n(x)$.

Then define:

$$\rho_\Sigma(x_1,...,x_N) = j_\phi^{\mu_1\ldots\mu_N}(x_1,...,x_N)\, n_{\mu_1}(x_1)\cdots n_{\mu_N}(x_N)$$



Probability currents

Fact: the integral

$$P(\Sigma) = \int_{\Sigma^N} d\sigma(x_1) \cdots d\sigma(x_N)\ \rho_\Sigma(x_1, ..., x_N)$$

is **conserved**, i.e., the same for all space-like hypersurfaces Σ.

These facts suggest:

Generalized Born rule

Let Σ be a space-like hypersurface. Then

$$\rho_\Sigma(x_1, ..., x_N) d\sigma(x_1) \cdots d\sigma(x_N)$$

is the probability to detect N particles in the infinitesimal 3-volumes $d\sigma(x_k)$ around $x_k \in \Sigma$, for $k = 1, ..., N$.

Note: This rule is still associated with the measurement problem. We need a **justification** of that rule by a relativistic theory which is free of this problem.

A relativistic version of the many-worlds interpretation

Relativistic MWI

We discuss a very simple version for N non-interacting (but entangeled) particles.

Basic ingredients:

1. **Multi-time wave equations**, e.g.:

$$(i\gamma_k^\mu \partial_{x_k^\mu} - m_k)\phi(x_1, ..., x_N) = 0, \quad k = 1, ..., N.$$

2. **Covariant matter density function:**

$$m^\mu(x) = \sum_{k=1}^{N} m_k \int_{\Sigma^{N-1}} d\sigma(x_1) \cdots \widehat{d\sigma(x_k)} \cdots d\sigma(x_N)$$
$$j_\phi^{\mu_1 \ldots \mu_k = \mu \ldots \mu_N}(x_1, ..., x_k = x, ..., x_N) \prod_{j \neq k} n_{\mu_j}(x_j).$$

(Fact: $m^\mu(x)$ is independent of the choice of space-like hypersurface Σ.)

Relativistic MWI

Then the theory works in the usual way.

How is non-locality implemented?
$\rightarrow$ intrinsic in the wave function (non-local object) which determines the matter density.

Status w.r.t. relativity: compatible without additional problems.
(Multi-time wave fns. are very useful for that.)

A relativistic version of GRWf

Relativistic GRWf

We discuss a model by R. Tumulka, 2006.[1]

Basic ingredients:

1. **Law for multi-time wave fn.**, e.g.

$$(i\gamma_k^\mu \partial_{x_k^\mu} - m_k)\phi(x_1, ..., x_N) = 0, \quad k = 1, ..., N,$$

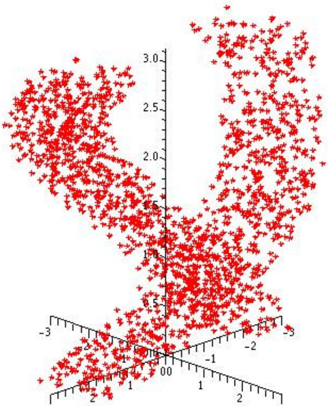
   interrupted from time to time by:

2. **Random collapses** which also generate **flashes** (discrete events in spacetime) which are also the collapse centers.

[1] J. Stat. Phys., 125:821-840, 2006.

Relativistic GRWf: ontology

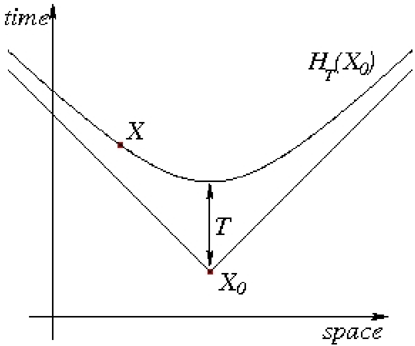
Picture of the world in GRWf: Objects are (sparse) galaxies of flashes in space-time.



Two moving extended objects in GRWf theory.

Relativistic GRWf: law for the flashes

1. Let N 'seed flashes' $X_1, ..., X_N \in \mathbb{R}^4$ be given.
2. Randomly determine N time differences $\Delta t_i \sim \exp(\lambda)$ and construct hyperboloids $H_{\Delta t_i}(X_i)$ of constant time-like distance Δt_i to X_i, $i = 1, ..., N$.



Picture credit: R. Tumulka

Relativistic GRWf: law for the flashes

3. Determine next generation of flashes Y_i, $i = 1, ..., N$ on the spacelike surfaces $\Sigma_i = H_{\Delta t_i}(X_i)$ at random according to the distribution

$$\mathrm{Prob}(Y_1 \in d\sigma_1, ..., Y_N) = \rho(y_1, ..., y_N) d\sigma_1 \cdots d\sigma_N$$

where

$$\rho(y_1, ..., y_N) = \int_{\prod_i \Sigma_i} d\sigma(z_1) \cdots d\sigma(z_N) \, |g_{\Sigma_1}(y_1, z_1)|^2 \cdots |g_{\Sigma_N}(y_N, z_N)|^2 \\ \times j_\phi^{\mu_1 \dots \mu_N}(z_1, ..., z_N) n_{\mu_1}(z_1) \cdots n_{\mu_N}(z_N)$$

with $g_\Sigma(y, z) = \mathcal{N}_\Sigma \exp\left(-\frac{\text{spacelike-dist}^2_\Sigma(y,z)}{2a^2}\right)$: covariant, normalized ($\mathcal{N}_\Sigma$) Gaussian on Σ.

Relativistic GRWf: law for the flashes

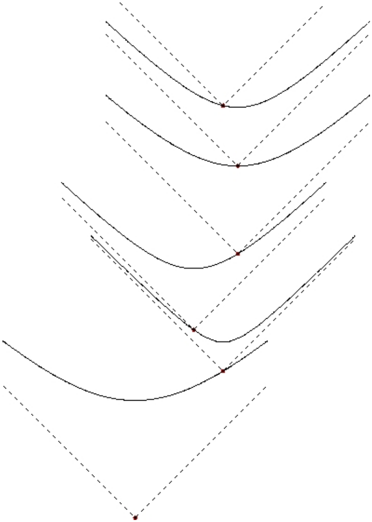
4. Next, collapse the wave fn. on $\prod_i \Sigma_i$ according to

$$\phi \to \phi'(z_1, ..., z_N) = \frac{g_{\Sigma_1}(Y_1, z_1) \cdots g_{\Sigma_N}(Y_N, z_N)\phi(z_1, ..., z_N)}{\rho^{1/2}(Y_1, .., Y_N)}$$

   and extend it again to the whole of $\mathbb{R}^{4N}$ using the multi-time equations.

5. Repeat the process starting with ϕ' and the new generation of flashes $Y_1, ..., Y_N$ as seed flashes.

Illustration[2] of the GRWf law for $N = 1$



[2] Picture credit: R. Tumulka

Relativistic GRWf: non-locality

Status of non-locality: Inherent in the construction process. Nature randomly determines a joint (and non-locally correlated) distribution of flashes. However, there is no fact about which flash determines which.

Status w.r.t. relativity: no preferred spacetime structure necessary to implement the non-locality!

A relativistic version of BM: the hypersurface Bohm-Dirac model

The hypersurface Bohm-Dirac model

Idea: Write down a Lorentz covariant law for actual world lines of particles, based on a multi-time wave function.

Main ingredients:

1. **Law for a multi-time wave fn.:** e.g.
$$(i\gamma_k^\mu \partial_{x_k^\mu} - m_k)\phi(x_1, ..., x_N) = 0, \quad k = 1, ..., N,$$
2. **Preferred folation of spacetime** into spacelike hypersurfaces $\Sigma_s,\ s \in \mathbb{R}$, normal vector field at folation: $n(x)$
3. **Law for the world lines:**

HBD guidance law

For $k = 1, ..., N$:

$$\frac{dX_k^\mu(s)}{ds} = j_\phi^{\mu_1 ... \mu_k = \mu ... \mu_N}(x_1, ..., x_N) \prod_{j \neq k} n_{\mu_j}(x_j)\Big|_{x_l = X_l(s),\ l=1,...,N}$$

HBD model: properties

- **Lorentz invariance:** if the foliation obeys a Lorentz invariant law, then the trajectories will be Lorentz covariant.
- **Equivariance** on the foliation: if the distribution of initial positions on one $\Sigma_{s_0} \in \mathcal{F}$ agrees with $\rho_{\Sigma_{s_0}}$, then the distribution of positions on all $\Sigma_s \in \mathcal{F}$ agrees with ρ_Σ.
- '$\rho = |\phi|^2$' nowhere else ('quantum equilibrium cannot hold in all Lorentz frames').
- **Unobservability of $\mathcal{F}$:** In measurements of position, one will nevertheless obtain the usual quantum statistics given by ρ_Σ for all spacelike Σ.

  Reason: measurements influence trajectories. To see that one gets the right statistics, note that one can always wait to read out result on a later surface in the foliation where agreement is ensured.

About the foliation

- $\mathcal{F}$ is a structure in addition to spacetime, not directly in conflict with it.
- Certain spacetimes have **natural folations,** e.g. in case a spacetime has a Big Bang, we can take as $\mathcal{F}$ the surfaces of constant time-like distance to the Big Bang.
- Important class of laws for the folation: one can **extract $\mathcal{F}$ from the (universal) wave fn.**
- Unclear which foliation to choose. (All foliations will lead to the same statistics in measurements, though not to the same trajectories.)
- **Light cones instead of foliation?** (Idea: light cones are no additional structure.) Need to use future light cones to get nonlocality. But: then the law has no equivariance property and it is unclear how to recover the quantum predictions.

Nonlocality in the HBD model

$$\frac{dX_k^\mu(s)}{ds} = j_\phi^{\mu_1 \ldots \mu_k = \mu \ldots \mu_N}(x_1, \ldots, x_N) \prod_{j \neq k} n_{\mu_j}(x_j) \Big|_{x_l = X_l(s),\ l=1,\ldots,N}$$

Explicit nonlocality: nonlocality is instantaneous along the surfaces Σ_s of the foliation. Nothing needs to travel between the particles to create this influence.

Nonlocality cannot be used for superluminal signaling: just as in normal QM it cannot be. In the HBD model, this is a consequence of quantum equilibrium, i.e., random distribution of positions according to ρ_Σ.

Comparison of the HBD model and rel. GRWf

GRWf does not need a preferred foliation. One reason is that as a stochastic theory, it does not need to specify how nonlocality is conveyed. This is different for the HBD model.

The difference can be pin-pointed by the following more abstract criterion:

Microscopic parameter independence (mPI)

Let A, B be spacelike separated regions in the future of a spacelike hypersurface Σ_0. Then, for given initial conditions on Σ_0, the choice of different external fields in B has no influence on the distribution of the primitive ontology in A.

GRWf respects mPI, the HBD model violates it.

Conclusions

- Multi-time wave functions are a useful concept to define Lorentz covariant laws for the primitive ontology.
- Precise versions of quantum theory show that non-locality can be fully compatible with relativity (see rel. GRWf, rel. MWI)
- In case of the HBD model, the determinism (and single world) makes it difficult to express non-locality without a foliation (at least if the quantum statistics should come out).
- Whether the HBD model can be accepted as 'relativistic' depends on the definition of 'relativistic'. Lorentz invariance is satisfied. But additional spacetime structure is used. (One can argue, though, that in a sense this spacetime structure exists anyway, e.g. in certain spacetimes, or in the wave function, and that the HBD model merely uses it).

Questions?