The Measurement Problem

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Overview

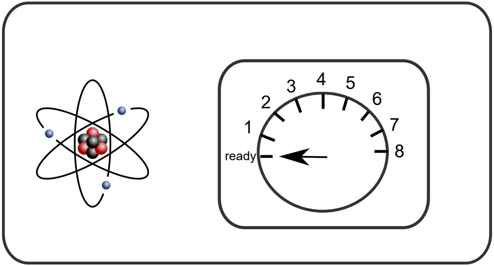
1. What is the measurement problem?
2. Paradox of Schrödinger's cat
3. Conclusions
4. Possible solutions
5. Common objections & responses
6. (Optional:) Wigner's friend

Assumptions

We will show that the following assumptions are incompatible with each other.

0. QM can be applied to every (isolated) physical system.
1. The wave function is a complete description of a physical system.
2. In each run of the experiment, there is a unique outcome.
3. The evolution of the wave function of an isolated system is given by Schrödinger's equation.

Modeling the measurement process



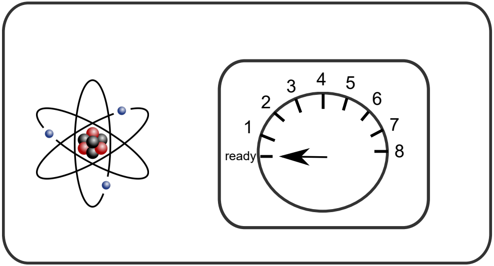
Initial object wave function: ψ_0

Initial apparatus wave function: Φ_0.

$\rightarrow$ Initial wave function of whole system: $\Psi(t_0) = \psi_0 \otimes \Phi_0$.

Experiment with N discrete outcomes $\alpha = 1, 2, ..., N$**:** ONB of object states $\psi_1, ..., \psi_N$; apparatus states $\Phi_1, ..., \Phi_N$ with macroscopically disjoint supports in configuration space.

Modeling the measurement process[1]



Schödinger time evolution (linear):

$$\Psi(t_0) = \psi_0 \otimes \Phi_0 \quad \longrightarrow \quad \sum_{\alpha=1}^{N} \psi_\alpha \otimes \Phi_\alpha = \Psi(t_1)$$

[1]Picture credit (atom):
https://de.wikipedia.org/wiki/Datei:Stylised_atom_with_three_Bohr_model_orbits_and_stylised_nucleus.svg

The measurement problem

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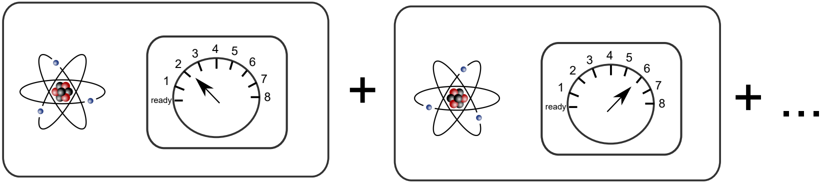
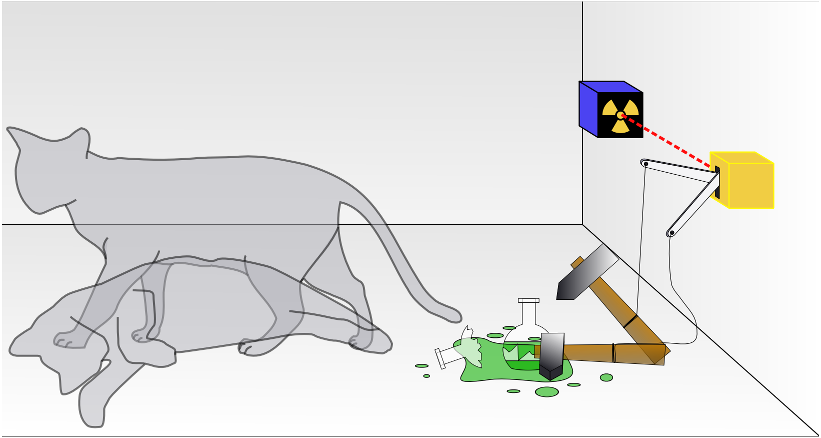


This naively leads to a superposition of states corresponding to different outcomes, **not** a random single definite outcome (as one observes). This discrepancy is called the **measurement problem.**

Schrödinger's cat[2]

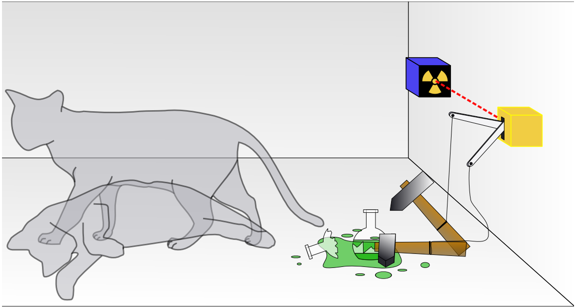


[2]Picture credit: https://en.wikipedia.org/wiki/Schr%C3%B6dinger%27s_cat#/media/File:Schrodingers_cat.svg

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Schrödinger's cat



$$\begin{aligned} \Psi(t_0) &= \psi_{\text{atom},\,0} \otimes \psi_{\text{counter, ready}} \otimes \psi_{\text{cat},\,0} \\ \longrightarrow \quad \Psi(t_1) &= \frac{1}{\sqrt{2}} \psi_{\text{atom, decayed}} \otimes \psi_{\text{counter, triggered}} \otimes \psi_{\text{cat, dead}} \\ &+ \frac{1}{\sqrt{2}} \psi_{\text{atom, not decayed}} \otimes \psi_{\text{counter, ready}} \otimes \psi_{\text{cat, alive}} \end{aligned}$$

Exactly analogous to the previous situation

Conclusion

(At least) one of the following assumptions must be incorrect:

0. QM can be applied to every (isolated) physical system.
1. The wave function is a complete description of a physical system.
2. In each run of the experiment, there is a unique outcome.
3. The evolution of the wave function of an isolated system is given by Schrödinger's equation.

Solving the measurement problem

... requires negating one of the conflicting assumptions (better not 0.).

Negating 1. (completeness) leads to Bohmian mechanics (BM). (Particles in addition to the wave fn.)

Negating 2. (unique outcomes) leads to the many worlds interpretation (MWI). (Every possible outcome occurs, but in a different world.)

Negating 3. (correctness of Schrödinger eq.) (and also (a)) leads to objective collapse theories. (Modification of Schrödinger's eq. by stochastic term which causes random, objective collapses.)

More about these theories in subsequent lectures.

Copenhagen interpretation

Uses the

Collapse postulate

If one observes the result α in the measurement of an observable A at t, the quantum state of the system afterwards jumps to the respective eigenstate ψ_α.

Discussion:

- Produces definite outcome by force.
- Overrides Schrödinger eq. (negates 3).
- Uses distinction between **classical observers**/measurement devices (which can collapse Ψ) and **quantum systems**. Quite arbitrary. Contradicts reductionism.
- Negates 0 (universal applicability of QM, observers do not obey QM).
- Negates 1 (wave fn. is complete description).

→ Many problems and inconsistencies. One can do much better.

Common excuses & responses

Many people deny that there is a measurement problem. We have collected some objections.

Excuse 1: Maybe we will really see superpositions of a dead and alive cat once technology progresses.

Response: But we do not now. This fact must be explained!

Excuse 2: Nobody can solve the Schödinger eq. for $N = 10^{23}$ particles.

Response: Correct. But for the argument it is enough to know what happens qualitatively . Linearity leads to a superposition of macroscopically different states.

Common excuses & responses

Excuse 3: Consciousness solves the measurement problem.

Response: No. That would amount to negating 0. (universality of QM) and 1. (completeness).

Challenge for anyone taking this view: Set up a consistent theory of consciousness.

Excuse 4: Systems are never really isolated.

Response: One can always take into account more, if need be the whole universe. If that is also not accepted, what hope for any theory remains?

Excuse 5: What if the initial wave fn. is not a product?

Response: It is not important that it is a product, just that one can perform a measurement on every initial ψ.

Excuse 6: The collapse of the wave fn. is just like he collapse of a probability distribution. When one obtains more information, say $X \in S$, then one has to update the prob. distr. ρ_t accoding to

$$\rho_{t^+}(x) \to 1_{x \in S}\, \rho_{t^-}(x).$$

Response: Striking parallel. However, if the wave fn. is supposed to be complete, there is never any new information external to the wave fn. (The thought refers to the problematic observer-system split.)

Common excuses & responses

Excuse 7: Decoherence solves the measurement problem.

Response: Decoherence means that we have

$$\psi_0 \otimes \Phi_0 \quad \longrightarrow \quad \sum_{\alpha=1}^{N} \psi_\alpha \otimes \Phi_\alpha$$

where the supports of Φ_α, Φ_β are macroscopically disjoint in configuration space and therefore cannot brought to interference anymore. (Common for systems with many d.o.f.)

This exactly leads to the paradox of Schödinger's cat. ($\psi_{\text{cat, dead}}$ and $\psi_{\text{cat, alive}}$ are states with these properties).

Common excuses & responses

But what about decoherence in the reduced density matrix formalism?

Fact: In decoherence situations, with a system S and an environment E we have that the **reduced density matrix**

$$\rho_{\mathrm{red}} = \mathrm{tr}_E \, |S, E\rangle\langle S, E|$$

becomes approximately **diagonal** while $|S, E\rangle$ is entangled. Same mathematical form as a **statistical density matrix**

$$\rho_{\mathrm{stat}} = \sum_\alpha c_\alpha \, |\alpha\rangle\langle\alpha|$$

which describes a situation where one randomly prepares the wave function of S in one of the states $|\alpha\rangle$ with probabilities c_α.

But: The meaning of these two objects is not the same. In the reduced density matrix case, the system has no own state, only the joint entangled state $|S, E(t)\rangle$. To make this clear, one refers to such a diagonal ρ_{red} as an **improper mixture**.

Wigner's friend

Consider the following situation: Wigner's friend (F) is observing Schrödinger's cat (C). Wigner (W) later checks whether the friend has found the cat dead or alive.

Quantum description: If we model the whole system as quantum-mechanical (as we should), we will obtain the following superposition in the end:

$$|\text{dead}\rangle_C \otimes |\downarrow\rangle_F \otimes |\downarrow\rangle_W \; + \; |\text{alive}\rangle_C \otimes |\uparrow\rangle_F \otimes |\uparrow\rangle_W$$

This is again a macroscopic superposition, and leads to another instance of the measurement problem.

Wigner's friend

If one invokes the collapse postulate of orthodox QM, then one obtains additional difficulties:

1. One part of the system (presumably (F) or (W)) is not modeled via a wave fn. ($\to$ problematic split of the world into quantum/classical)
2. $\to$ Several possible wave functions:
   (a) $\psi_{CF} = |\text{dead}\rangle_C \otimes |\downarrow\rangle_F + |\text{alive}\rangle_C \otimes |\uparrow\rangle_F$,
   (b) $\psi_{CW} = |\text{dead}\rangle_C \otimes |\downarrow\rangle_W + |\text{alive}\rangle_C \otimes |\uparrow\rangle_W$
   (c) $\psi_{FW} = |\downarrow\rangle_F \otimes |\downarrow\rangle_W + |\uparrow\rangle_F \otimes |\uparrow\rangle_W$
3. Is one of these ψ's the right one, or are several possible? If so, how are they related?
4. Who can collapse a wave function? (C, F or W? One of them? All of them?)
5. When does a collapse occur? (When C dies/lives, when F gets to know, or when W gets to know?)

$\to$ Again all kinds of problems and inconsistencies with the collapse postulate.

Conclusions

- QM faces the measurement problem: either 1. the wave fn. description is **not complete**, or 2. there are **no unique outcomes** or 3. **Schödinger's eq. is not correct.**
- Negating one of these assumptions leads to 1. **Bohmian mechanics**, or 2. **many worlds**, or 3. **collapse theories**.
- Schödinger's cat is an instance of the same general problem, not a 'quantum curiosity'.
- The collapse postulate in the Copenhagen interpretation does really solve the measurement problem but rather leads to further problems and inconsistencies.
- Numerous objections to the measurement problem do not apply. (Discussion: Do you have further ones...? ;-))

Questions?