THE DE BROGLIE-BOHM THEORY AS A RATIONAL COMPLETION OF QUANTUM MECHANICS

SCHOOL ON PARADOXES IN QUANTUM PHYSICS

JOHN BELL INSTITUTE FOR THE FOUNDATIONS OF PHYSICS

Hvar, Croatia

2 SEPTEMBER 2019

Jean BRICMONT

Institut de Recherche en Mathématique et Physique

UCLouvain

BELGIUM

WHAT IS THE MEANING OF THE WAVE FUNCTION?

IN ORTHODOX QUANTUM MECHANICS

$\Psi =$ STATE: VECTOR IN A HILBERT SPACE, e.g. $L^2(\mathbb{R}^N)$.

WHICH EVOLVES IN TIME:

$$\Psi_0 \longrightarrow \Psi_t = U(t)\Psi_0$$

$U(t)$ UNITARY OPERATOR

$=$ SOLUTION OF SCHRÖDINGER'S EQUATION

A= AN “OBSERVABLE” = SELF-ADJOINT OPERATOR ACTING ON THAT HILBERT SPACE. IF A HAS A BASIS OF EIGENVECTORS:

$A\Psi_i = \lambda_i \Psi_i$

WRITE Ψ IN THAT BASIS

$\Psi = \sum_i c_i \Psi_i \qquad \text{with} \qquad \sum_i |c_i|^2 = 1.$

THEN, WE HAVE THE BORN RULE ABOUT PROBABILITIES OF RESULTS OF MEASUREMENTS

$$\mathrm{P}(\text{Result} = \lambda_i \text{ when measure A, if state} = \Psi)$$
$$= |c_i|^2$$

AFTER THAT, THE STATE JUMPS OR IS REDUCED OR COLLAPSES TO Ψ_i.

The meaning of Ψ comes only from measurements. As John Bell puts it:

> It would seem that the theory is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a Ph D?

A DEEPER PROBLEM.

Existential angst: Am I a vector Ψ in a Hilbert space? Which is, in part, simply a function defined on a high-dimensional space, such as $\Psi \in L^2(\mathbb{R}^N)$?

What about you?

That is why I don't agree with the notion that the quantum state **even could** be a "complete description" of a physical system.

More basic problem than the one of "measurement".

Once you think about it, it is quite obvious that ordinary quantum mechanics cannot be the complete story. It does predict results of measurements, very accurately, but does not say anything about what is going on in the world.

We need an "ontology" or "beables" (Bell's word), namely we need to postulate something that exists outside of laboratories and that is not just the quantum state.

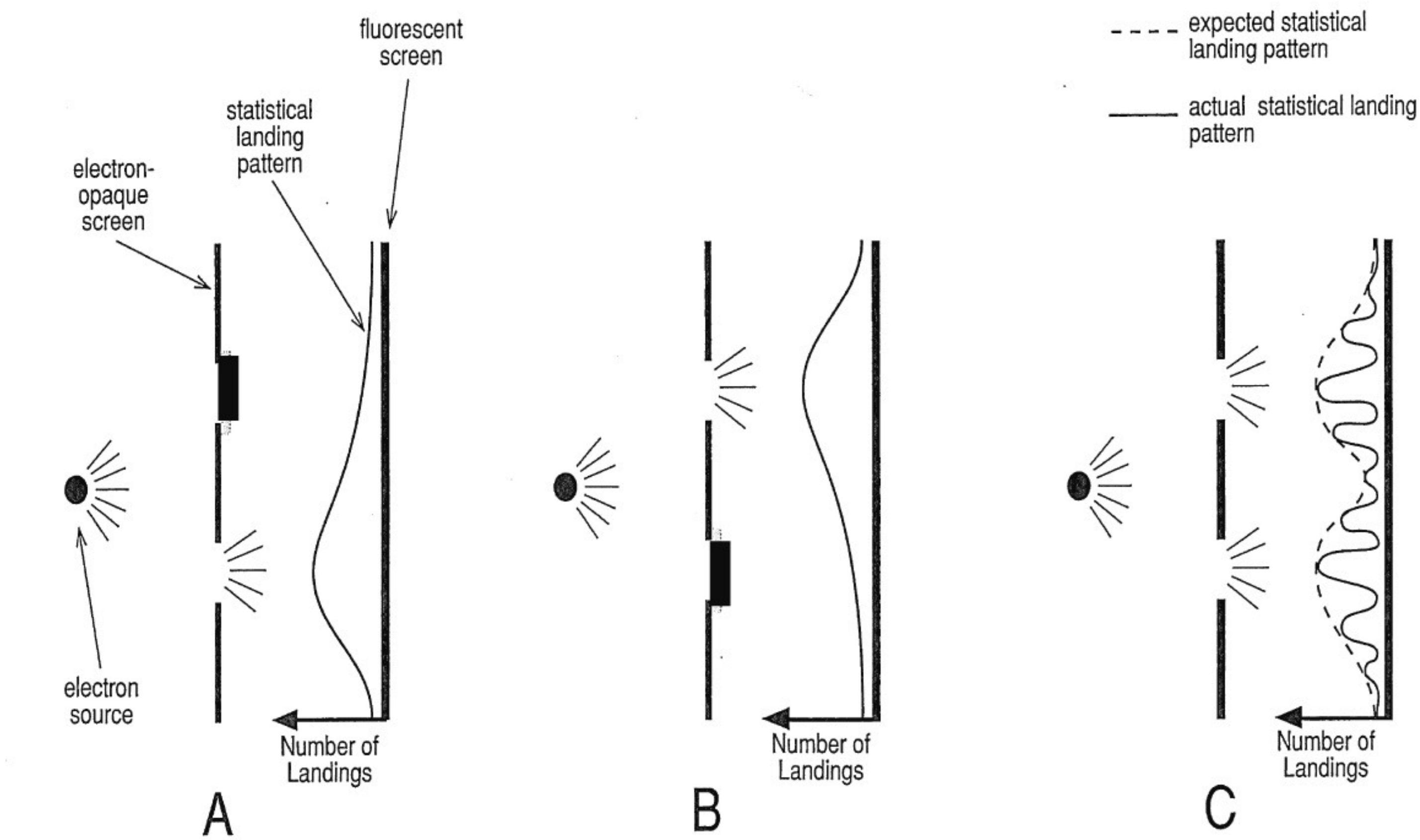
These beables are sometimes called "hidden variables" in the quantum literature, which is taken to be a negative word (by people who do not realize that there are problems with orthodox quantum mechanics).

THE DE BROGLIE-BOHM THEORY

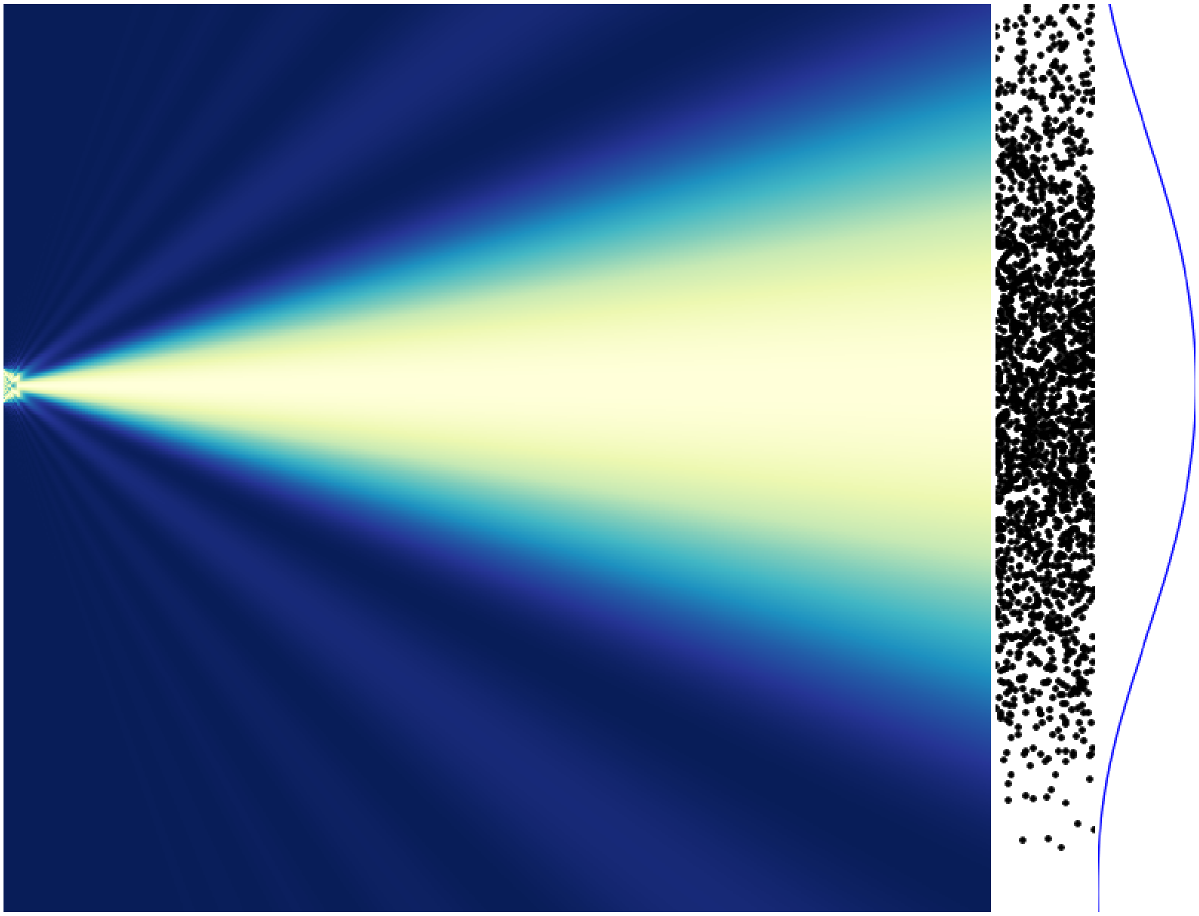
The theory of de Broglie (1927) and Bohm (1952), (also of Bell, Dürr, Goldstein, Zanghì):

1. Is a theory of "hidden variables" (although they are not at all hidden),
2. That accounts for all the phenomena predicted by ordinary quantum mechanics,
3. That explains why measurements do not in general measure pre-existing properties of a system (in other words, it explains why measuring devices have an "active role"),

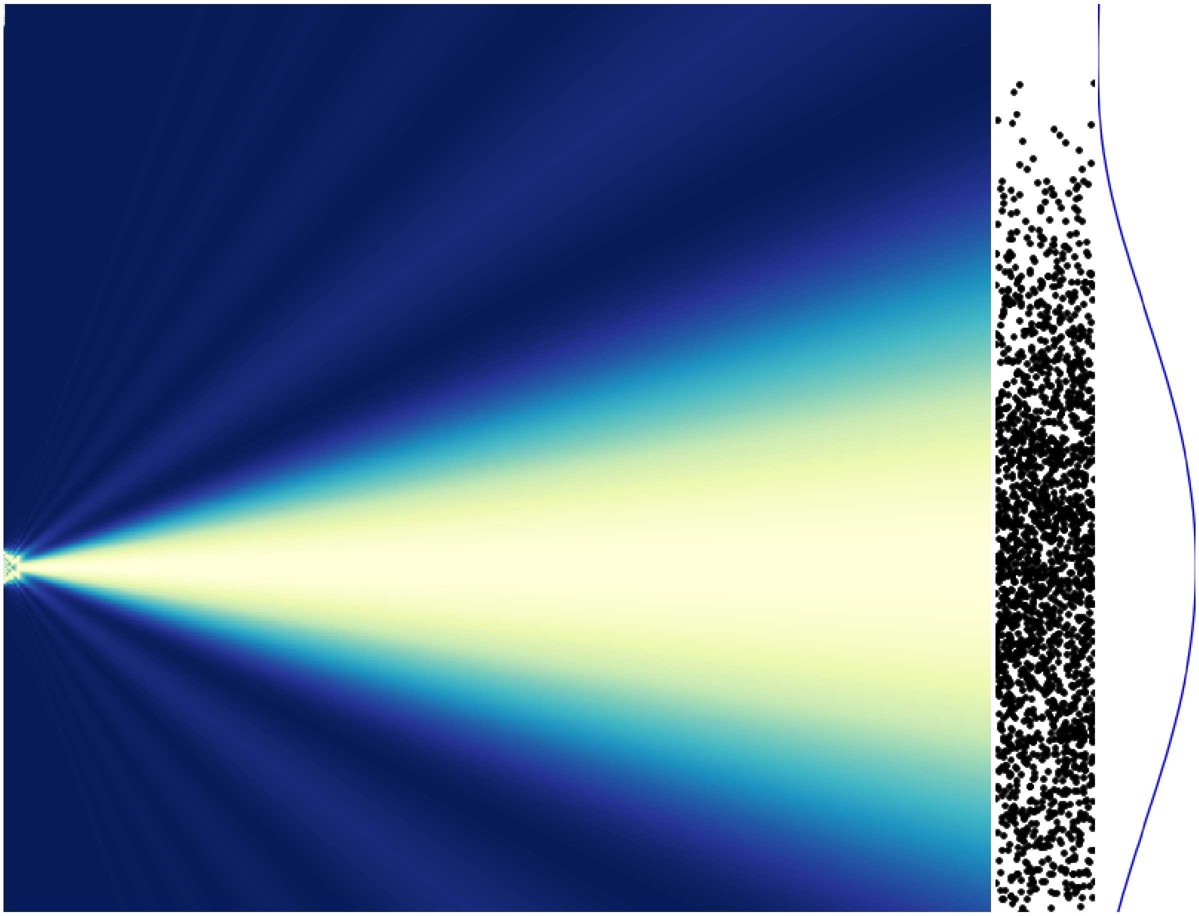
LET US THINK OF THE DOUBLE SLIT EXPERIMENT



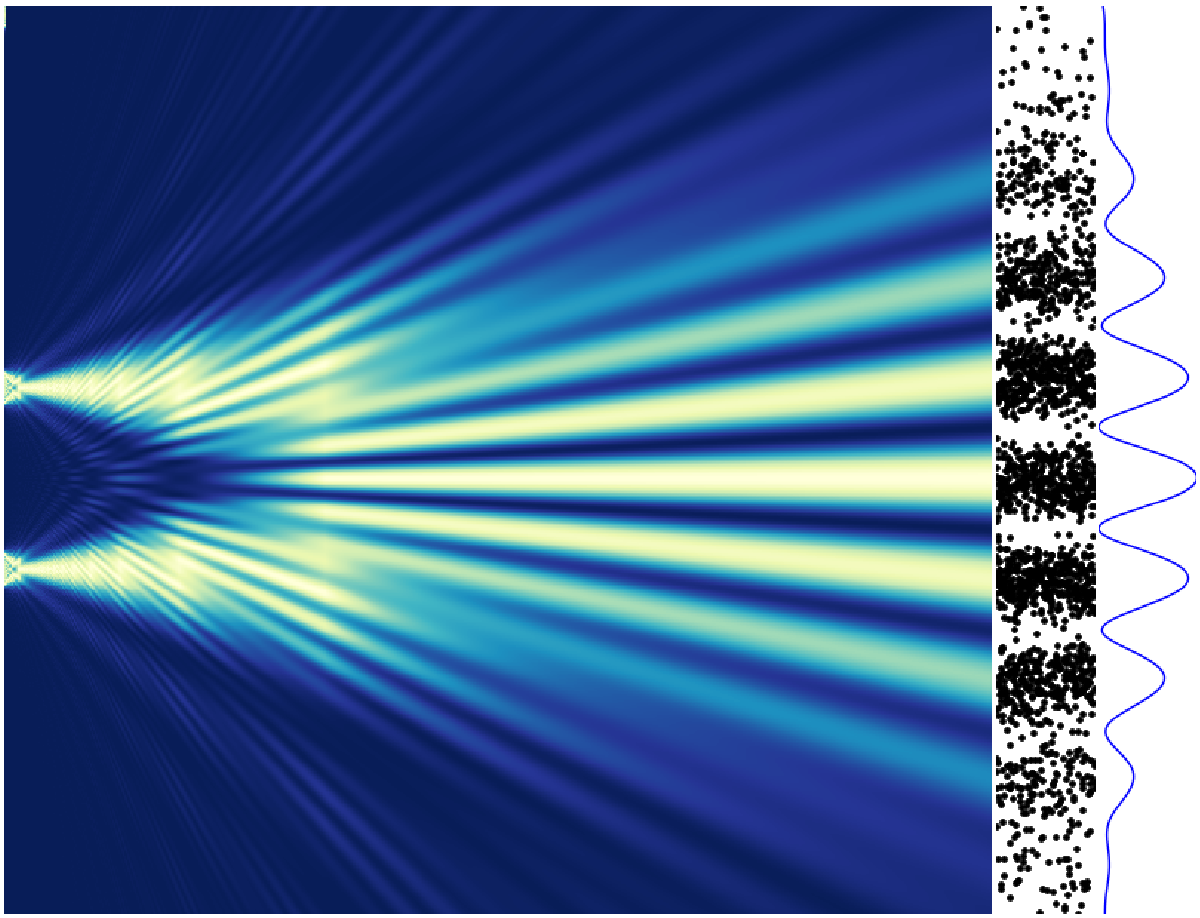
OR IN IMAGES:



INTENSITY OF THE FLOW OF PARTICLES WHEN ONLY THE UPPER SLIT IS OPEN



INTENSITY OF THE FLOW OF PARTICLES WHEN ONLY THE LOWER SLIT IS OPEN



INTENSITY OF THE FLOW OF PARTICLES WHEN BOTH SLITS ARE OPEN

HOW CAN ELECTRONS BE BOTH PARTICLES AND WAVES?

ELEMENTARY MY DEAR BOHR!

THEY ARE PARTICLES *GUIDED* BY WAVES.

THE BROGLIE-BOHM THEORY

In the de Broglie-Bohm's theory, the state of system is a pair (X, Ψ), where $X = (X_1, \ldots, X_N)$ denotes the actual positions of all the particles in the system under consideration.

$\Psi = \Psi(x_1, \ldots, x_N)$ is the usual quantum state, $(x_1, \ldots, x_N)$ denoting the arguments of the function Ψ.

X are the "hidden variables" in this theory; this is obviously a misnomer, since particle positions are the only things that we ever directly observe: think of the double-slit experiment for example, but we will see that this is true for **ANY** measurement.

The dynamics of the de Broglie-Bohm's theory is as follows: both objects Ψ and X evolve in time:

1. SCHRÖDINGER'S EQUATION.

For the quantum state, at all times, and whether one measures something or not:

$\Psi_0 \to \Psi_t = U(t)\Psi_0$

$i\partial_t \Psi(x_1, \ldots, x_N, t) = (H\Psi)(x_1, \ldots, x_N)$

(with $\hbar = 1$ and all masses $= 1$)

where H is the Hamiltonian:

$$H = -\frac{1}{2}\Delta + V,$$

and V is the potential.

THE QUANTUM STATE NEVER COLLAPSES.

2. GUIDING EQUATION:

The evolution of the positions is guided by the quantum state: writing $\Psi = Re^{iS}$

$$\frac{d}{dt}X_k(t) = \nabla_k S(X_1(t), \ldots, X_N(t))$$

for $k = 1, \ldots, N$, where $X_1(t), \ldots, X_N(t)$ are the actual positions of the particles at time t.

This can also be written as

$$\frac{d}{dt}X_k(t) = \frac{\mathrm{Im}(\Psi^* \cdot \nabla_k \Psi)}{\Psi^* \cdot \Psi}(X_1(t), \dots, X_N(t), t)$$

$\forall k = 1, \dots, N.$

This latter version can be generalized to particles with spin, by letting the $\cdot$ in $\Psi^* \cdot \nabla_k \Psi$ and in $\Psi^* \cdot \Psi$ stand for the scalar product between the spin components.

Of course, in that case, we have to replace Schrödinger's equation by Pauli's or Dirac's equation $\rightarrow$ other talks.

Note that in the equation:

$$\frac{d}{dt}X_k(t) = \frac{\mathrm{Im}(\Psi^* \cdot \nabla_k \Psi)}{\Psi^* \cdot \Psi}(X_1(t), \ldots, X_N(t), t),$$

the numerator of the RHS is the "quantum probability current", $j_k^Q = \mathrm{Im}(\Psi^* \cdot \nabla_k \Psi)$:

From Schrödinger's equation, we have

$$\frac{\partial}{\partial t}\Psi^* \cdot \Psi(t) = 2\mathrm{Re}(\Psi^* \cdot (-iH)\Psi)(t)$$

$$= \mathrm{Im}(-\Psi^* \cdot \Delta\Psi + 2\Psi^* \cdot V\Psi)(t) = -\mathrm{Im}(\Psi^* \cdot \Delta\Psi)(t)$$

$$= -\sum_{k=1}^{N} \nabla_k j_k^Q$$

since $\nabla_k \Psi^* \cdot \nabla_k \Psi = |\nabla_k \Psi|^2$ is real.

This can be summarized in the fundamental formula

$$\frac{d}{dt}X_k(t) = V_k^P(t) = \frac{j_k^Q(t)}{\rho^Q(t)}$$

with $\rho^Q(t) = \Psi^* \cdot \Psi(t) = |\Psi|^2$.

In other words, the velocity of the particles $V^P(t)$ in the de Broglie-Bohm's theory is the ratio of the "quantum probability current" $j^Q(t)$ and the quantum "probability density" $\rho^Q(t)$.

Double slit experiment: numerical solution in the de Broglie-Bohm theory.

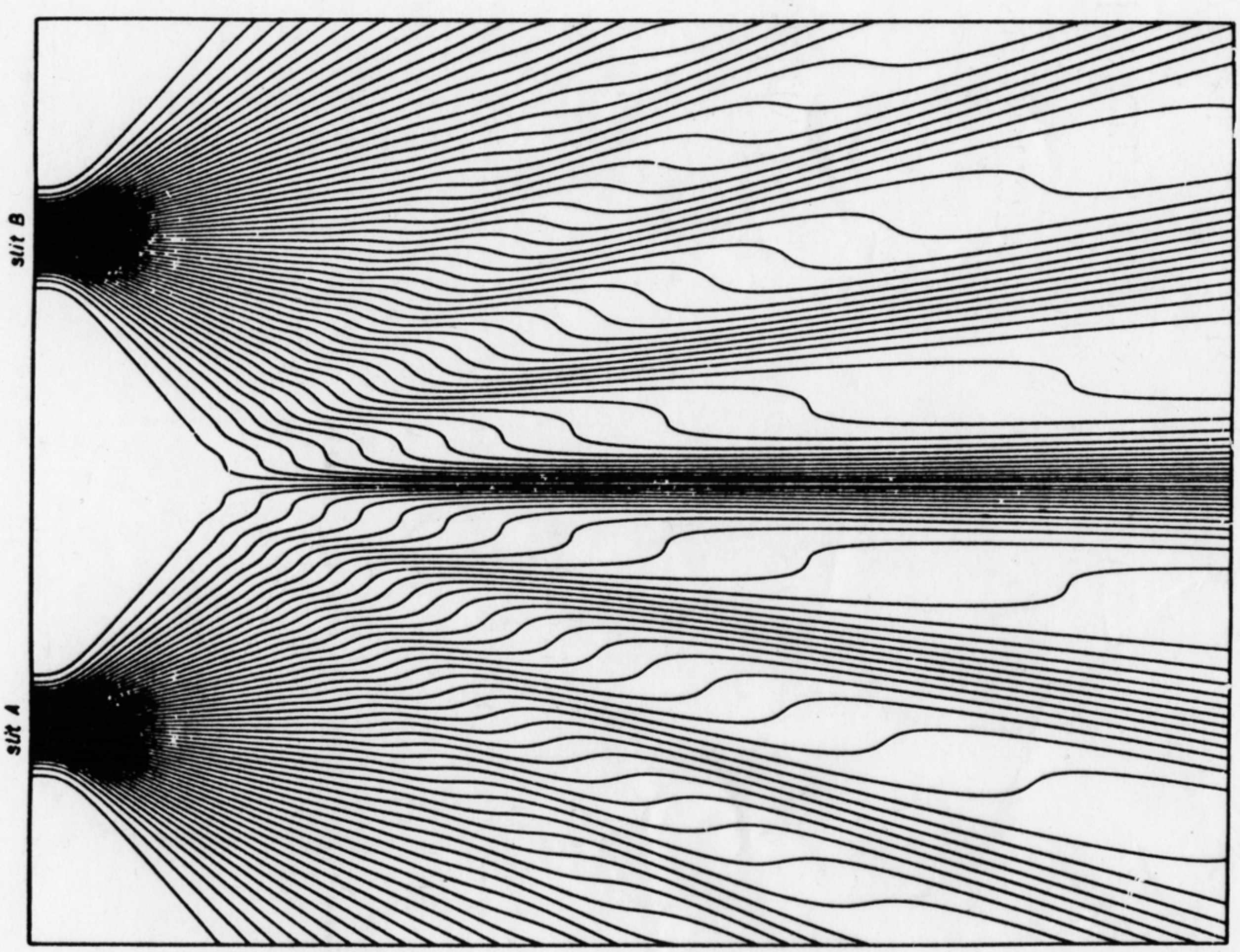
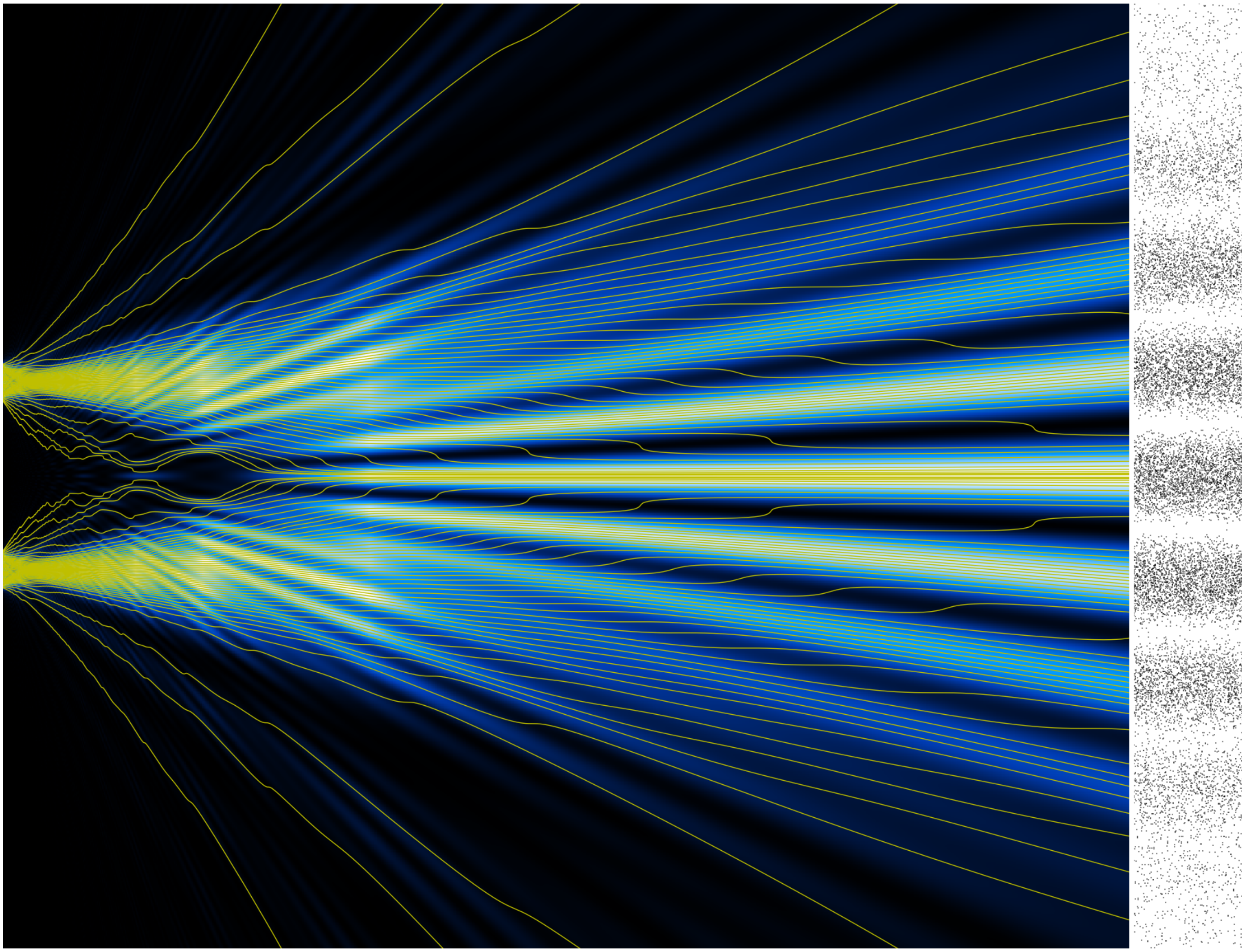


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

Motion *in vacuum* highly *non classical*!! Note that one can determine a posteriori through which hole that particle went !



INTENSITY OF THE FLOW OF PARTICLES WHEN BOTH SLITS ARE OPEN WITH ONE HUNDRED TRAJECTORIES SIMULATED

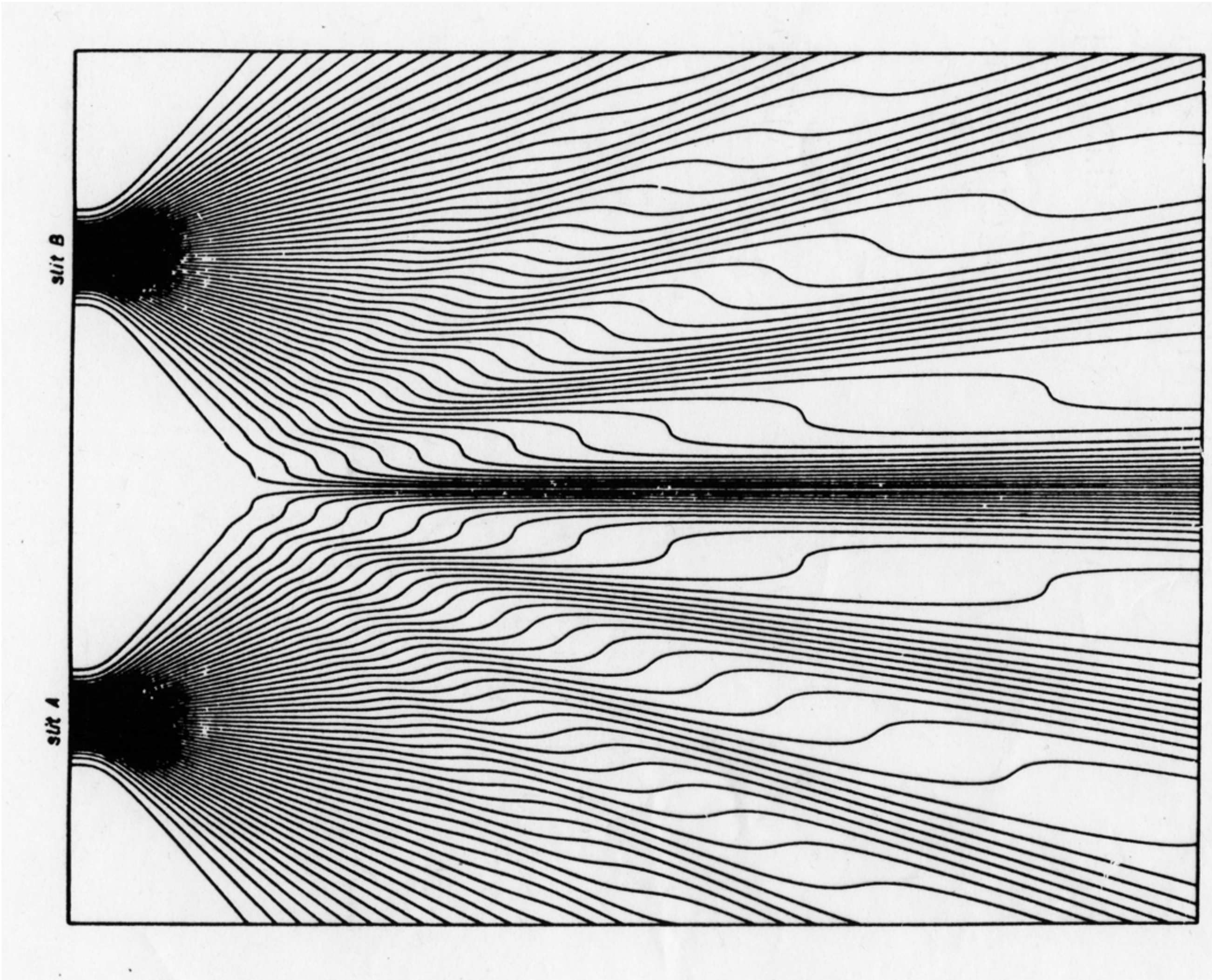
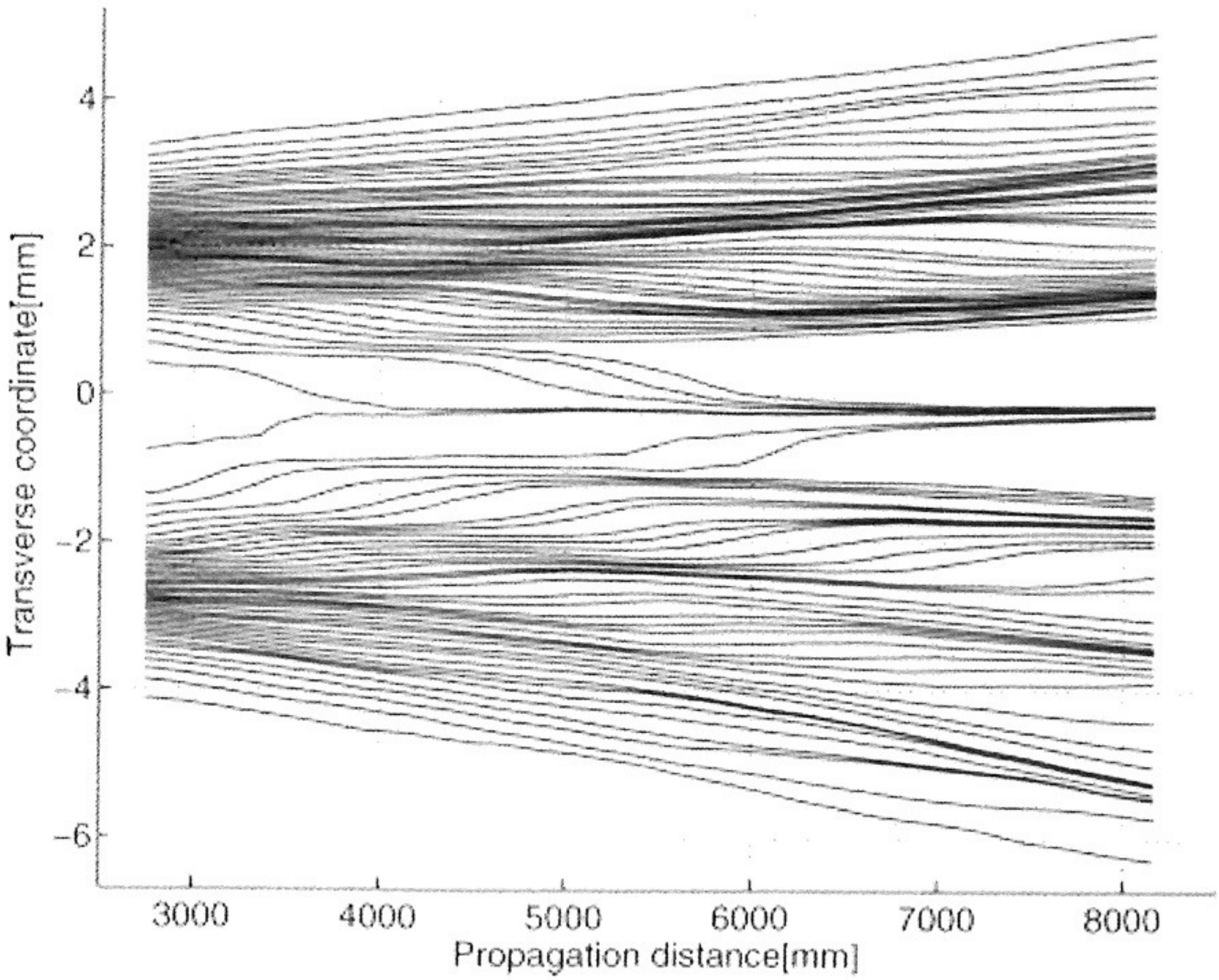


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

Note also the presence of a nodal line: by symmetry of Ψ, the velocity is tangent to the middle line; thus, particles cannot cross it.

Related experiment (Science, june 2011).



J. BELL:

It is not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes.

And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.

J. BELL

FOUR QUESTIONS:

- WHAT ABOUT THE STATISTICAL PREDICTIONS ?

- WHAT ABOUT THE "QUANTUM MEASUREMENTS" ?

- WHAT ABOUT THE COLLAPSE OF THE WAVE FUNCTION ?

- WHAT ABOUT (NON)-LOCALITY?

$\rightarrow$ **later talk.**

HOW DOES THE THEORY OF DE BROGLIE-BOHM ACCOUNT FOR THE STATISTICAL PREDICTIONS OF QUANTUM MECHANICS?

THANKS TO EQUIVARIANCE:

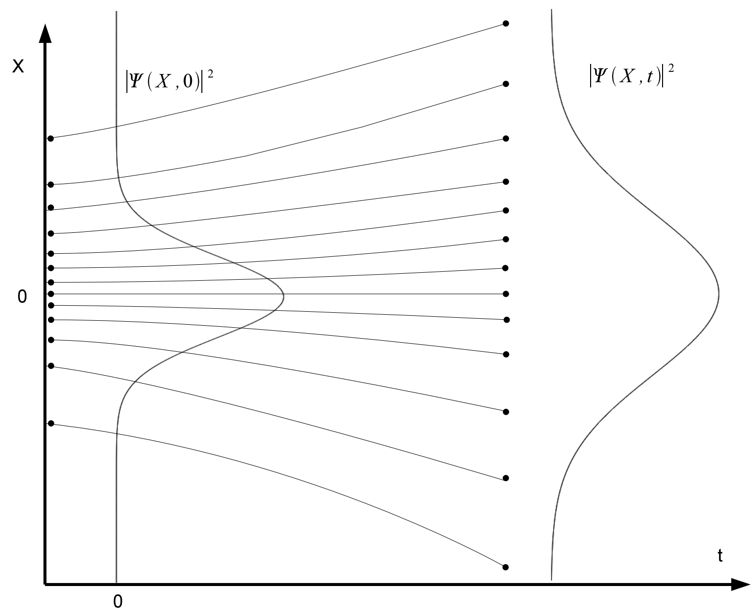
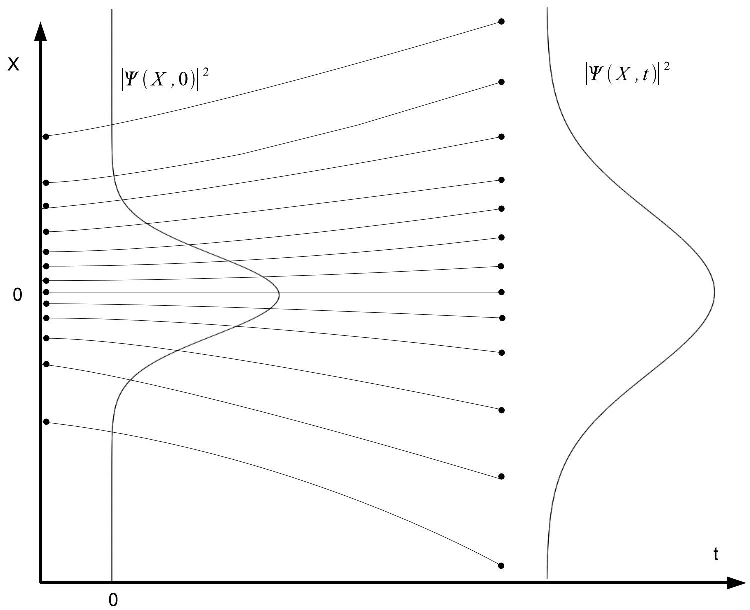
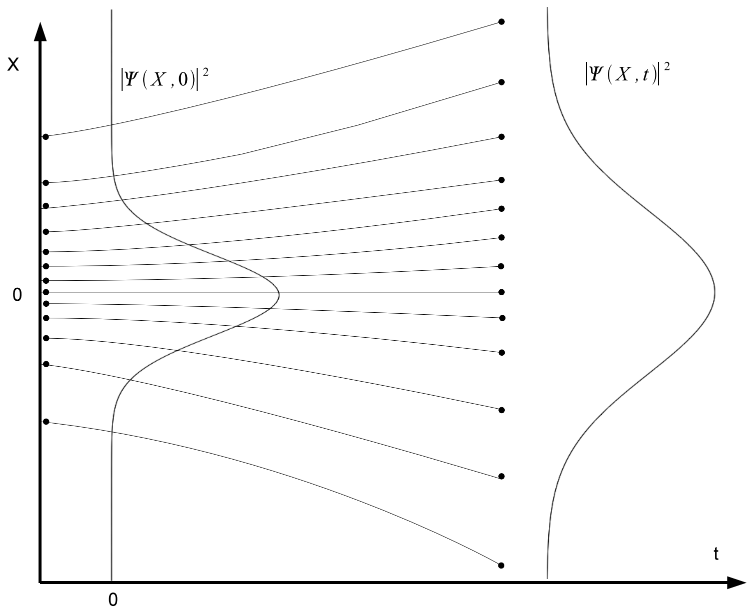


Illustration of the property of equivariance of the $|\Psi(X,t)|^2$ distribution, in one dimension, for a Gaussian Ψ. Each dot represents the position of a particle, both at time 0 and at time t, connected by trajectories.



The initial density of particles ρ_0^P is (approximately) given by $\rho_0^P(X) = |\Psi(X,0)|^2$, see the left of the picture.



Then, it is an easy consequence of the de Broglie-Bohm's theory that the empirical density of particles at later times $\rho_t^P(x)$ will satisfy $\rho_t^P(x) = |\Psi(x,t)|^2$, where $\Psi(x,t)$ is the solution of the Schrödinger equation and $\rho_t^P(x)$ comes from the guiding equation:

$\frac{d}{dt}X(t) = \nabla S(X(t),t)$,

with $\Psi(x,t) = R(x,t)e^{iS(x,t)}$.

See the right of the picture.

EQUIVARIANCE FOLLOWS FROM WHAT WE SHOWED PREVIOUSLY:

$$\frac{d}{dt}X(t) = V_k^P(X,t) = \frac{j_k^Q(X,t)}{\rho^Q(X,t)}$$

with $\rho^Q(X,t) = \Psi^* \cdot \Psi(X,t) = |\Psi(X,t)|^2$ and

$$\frac{\partial}{\partial t}\rho^Q(X,t) = -\operatorname{div}(j_k^Q(X,t))$$

For a fluid of density $\rho(x,t)$ and velocity field $V(x,t)$, one has the continuity equation:

$$\frac{\partial}{\partial t}\rho(x,t) = -\operatorname{div}(V(x,t)\rho(x,t)).$$

So, if we put here $\rho(x,t) = \rho^P(x,t)$ and $V(x,t) = V^P(x,t) = \frac{j_k^Q(x,t)}{\rho^Q(x,t)}$, we get that the identity

$$\rho^P(x,t) = \rho^Q(x,t) = |\Psi(x,t)|^2$$

is preserved in time.

SO, IF WE ASSUME THAT $\rho_0 = |\Psi_0|^2$ AT SOME INITIAL TIME, IT WILL HOLD AT ALL TIMES.

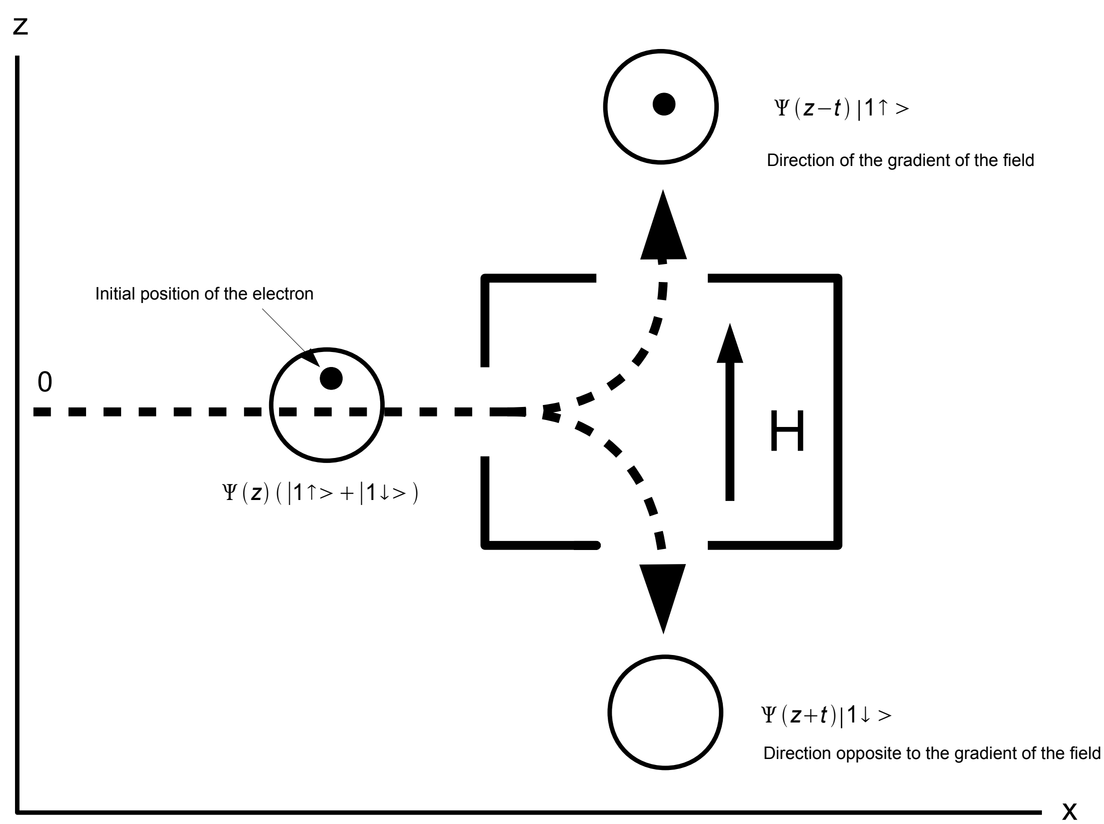
THE STATISTICAL PREDICTIONS OF QUANTUM MECHANICS ARE RECOVERED, AT LEAST AS FAR AS THE POSITIONS OF THE PARTICLES ARE CONCERNED.

THE ASSUMPTION THAT $\rho_0 = |\Psi_0|^2$ IS CALLED QUANTUM EQUILIBRIUM.

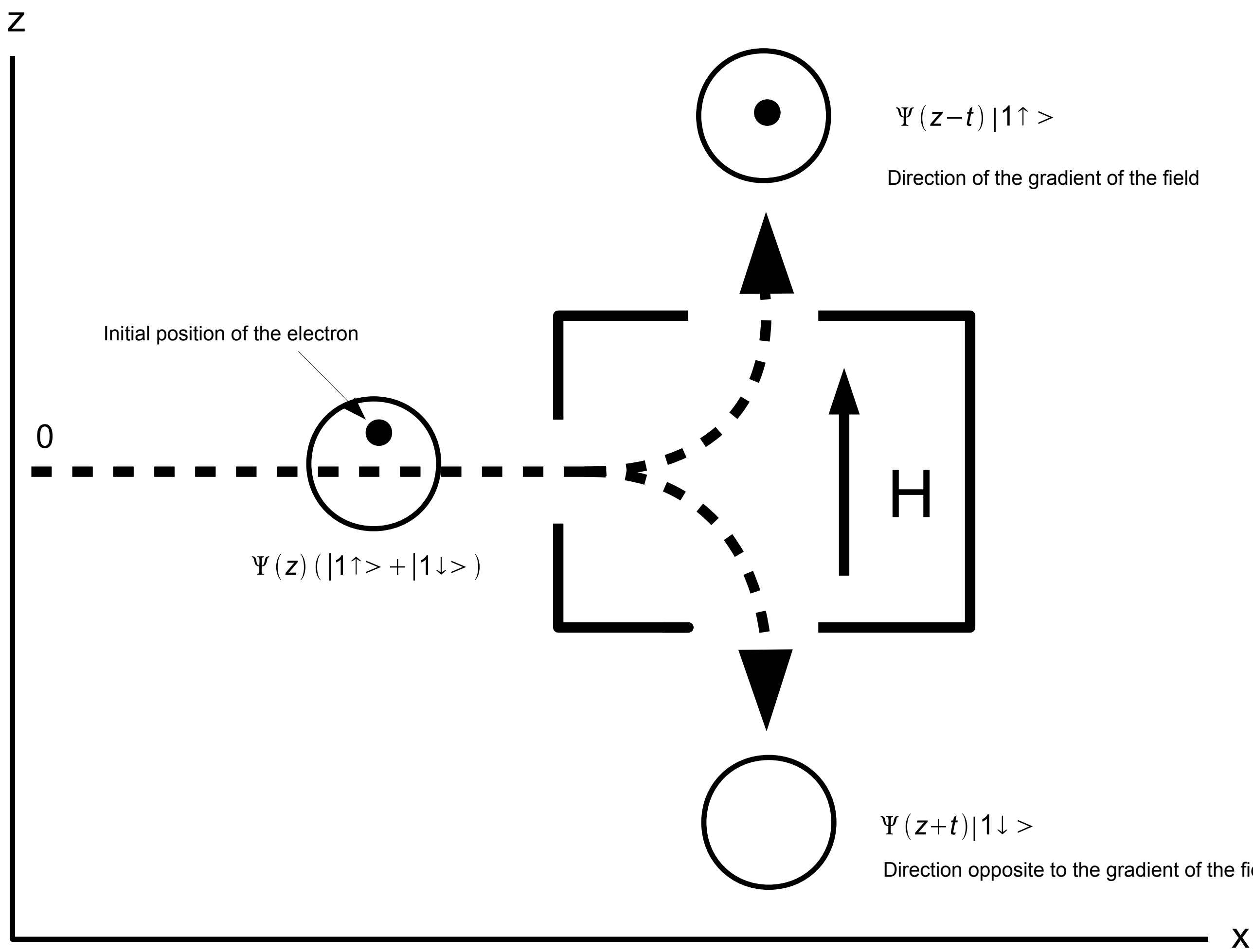
BUT SINCE EVERY OTHER “MEASUREMENT” IS ULTIMATELY A MEASUREMENT OF POSITION (AS WE WILL SEE) WE WILL ALSO RECOVER THE QUANTUM PREDICTIONS FOR THOSE “MEASUREMENTS”.

HOW DOES THE DE BROGLIE-BOHM THEORY ACCOUNT FOR THE "MEASUREMENT" OF SPIN ?

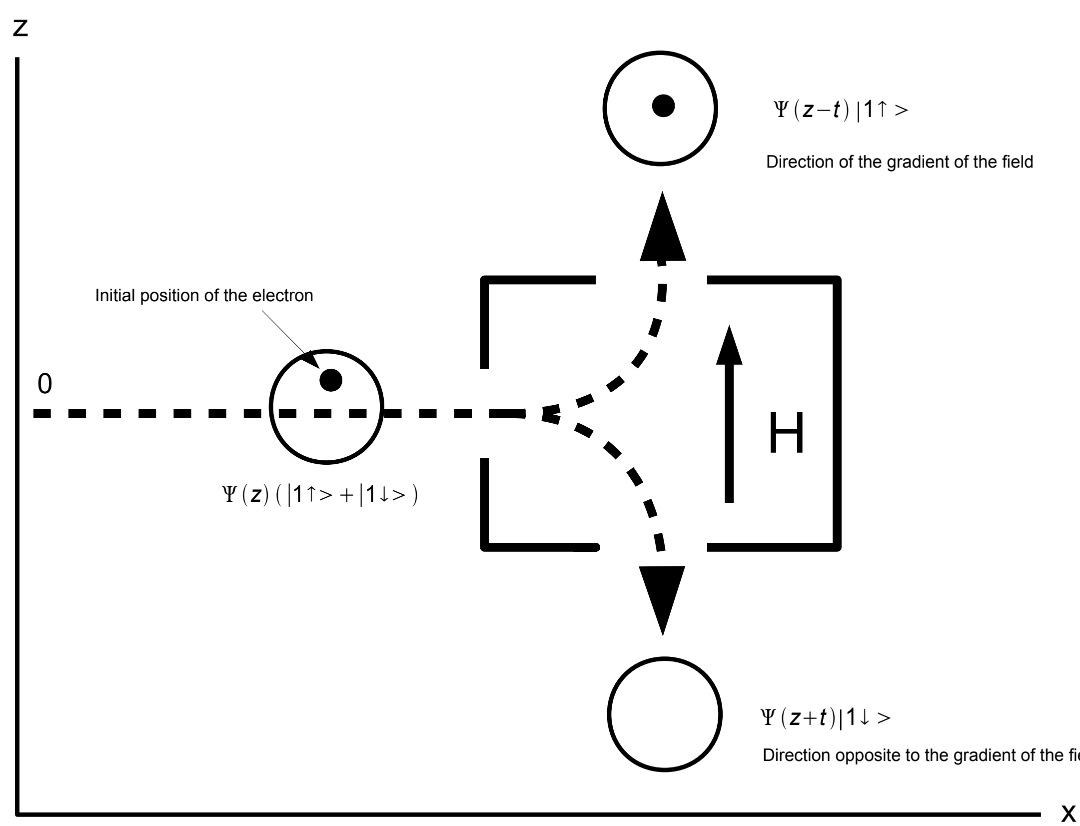
Consider a Stern-Gerlach apparatus "measuring" the spin. Let H be the magnetic field. The arrow in the picture indicates the direction of the gradient of that field.



The $|1\uparrow>$ part of the state always goes in the direction of the gradient of the field, and the $|1\downarrow>$ part always goes in the opposite direction.



But if the particle is initially in the upper part of the support of the wave function (for a symmetric wave function), it will always go upwards. That is because there is a nodal line in the middle of the figure that the particles cannot cross.



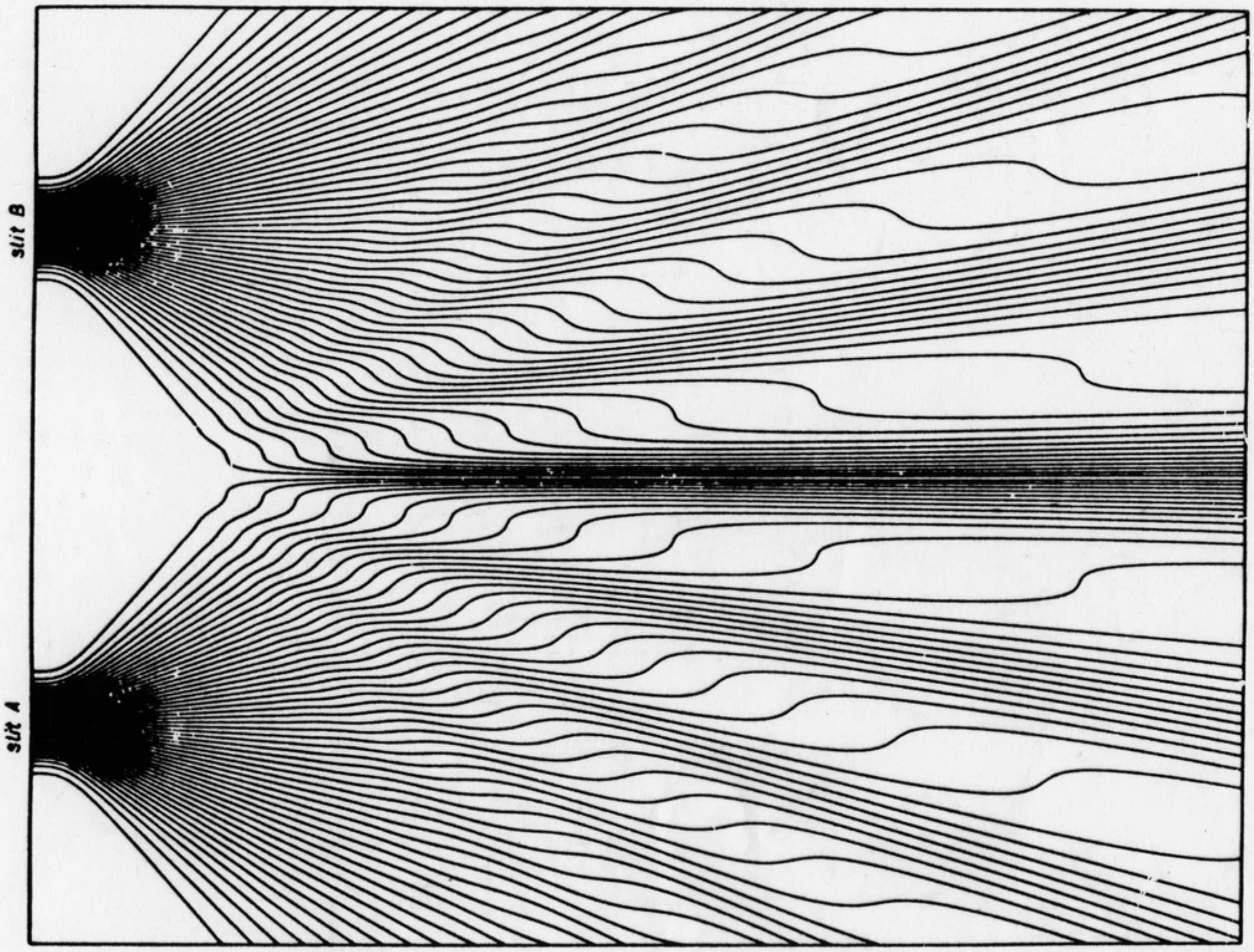
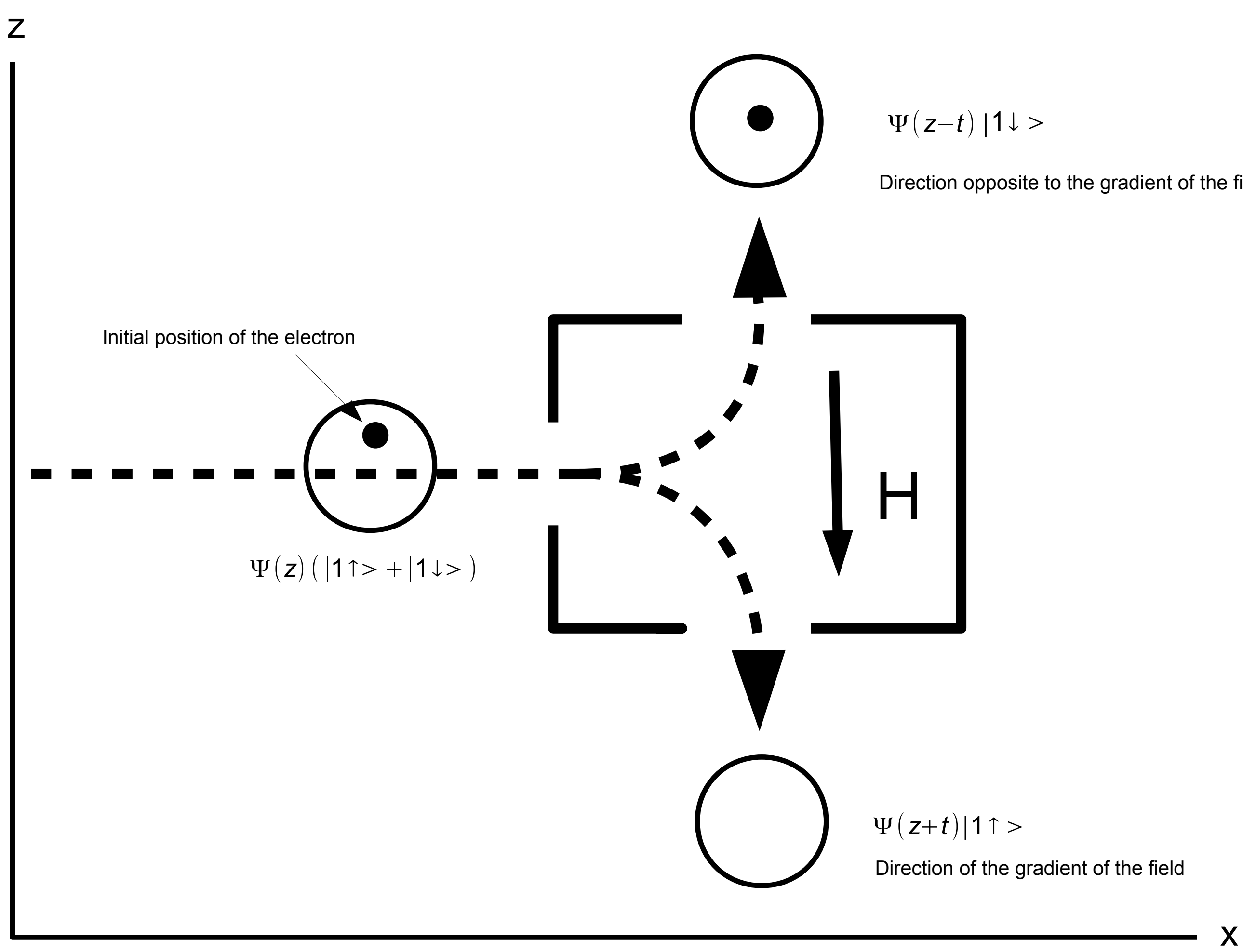


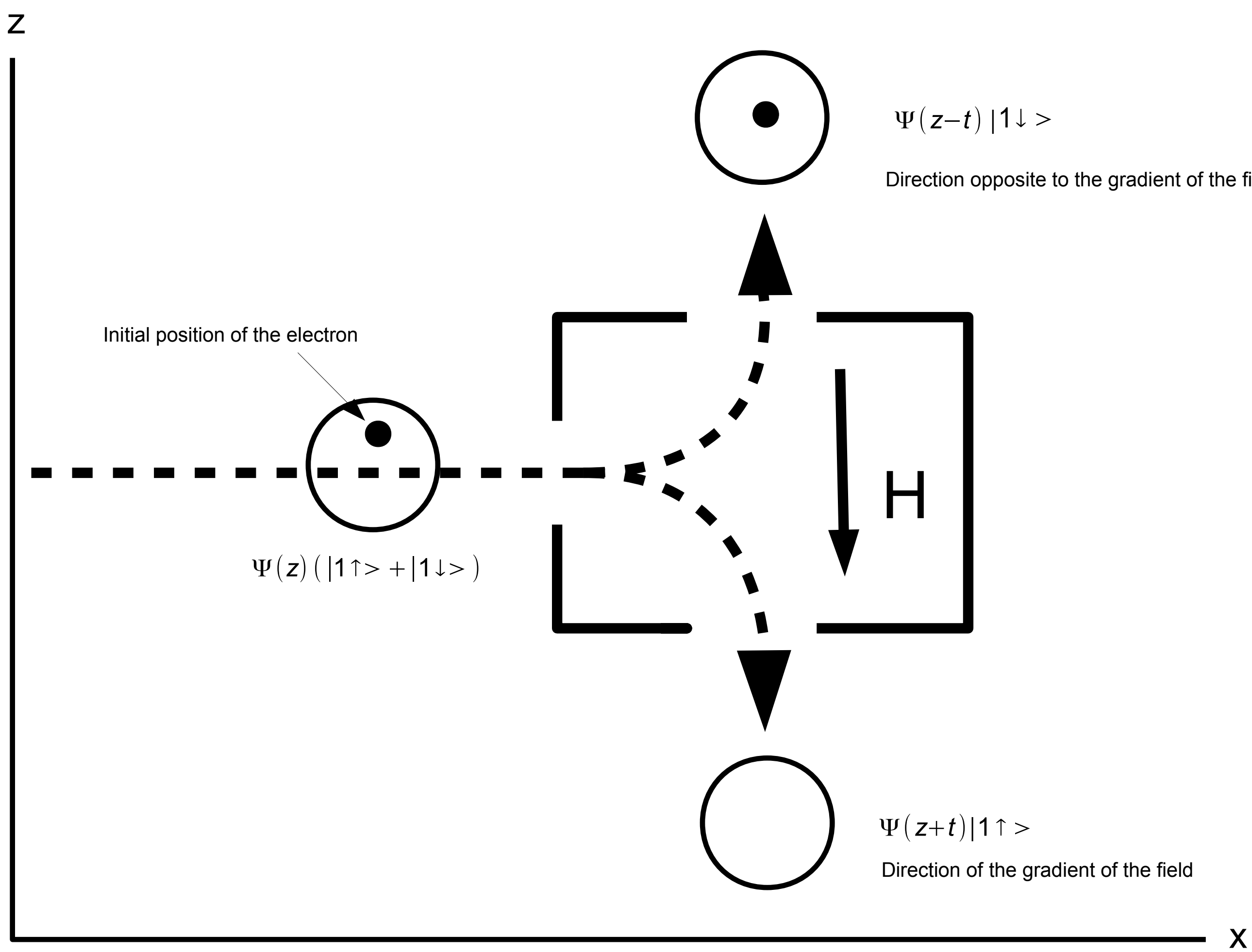
Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

as here

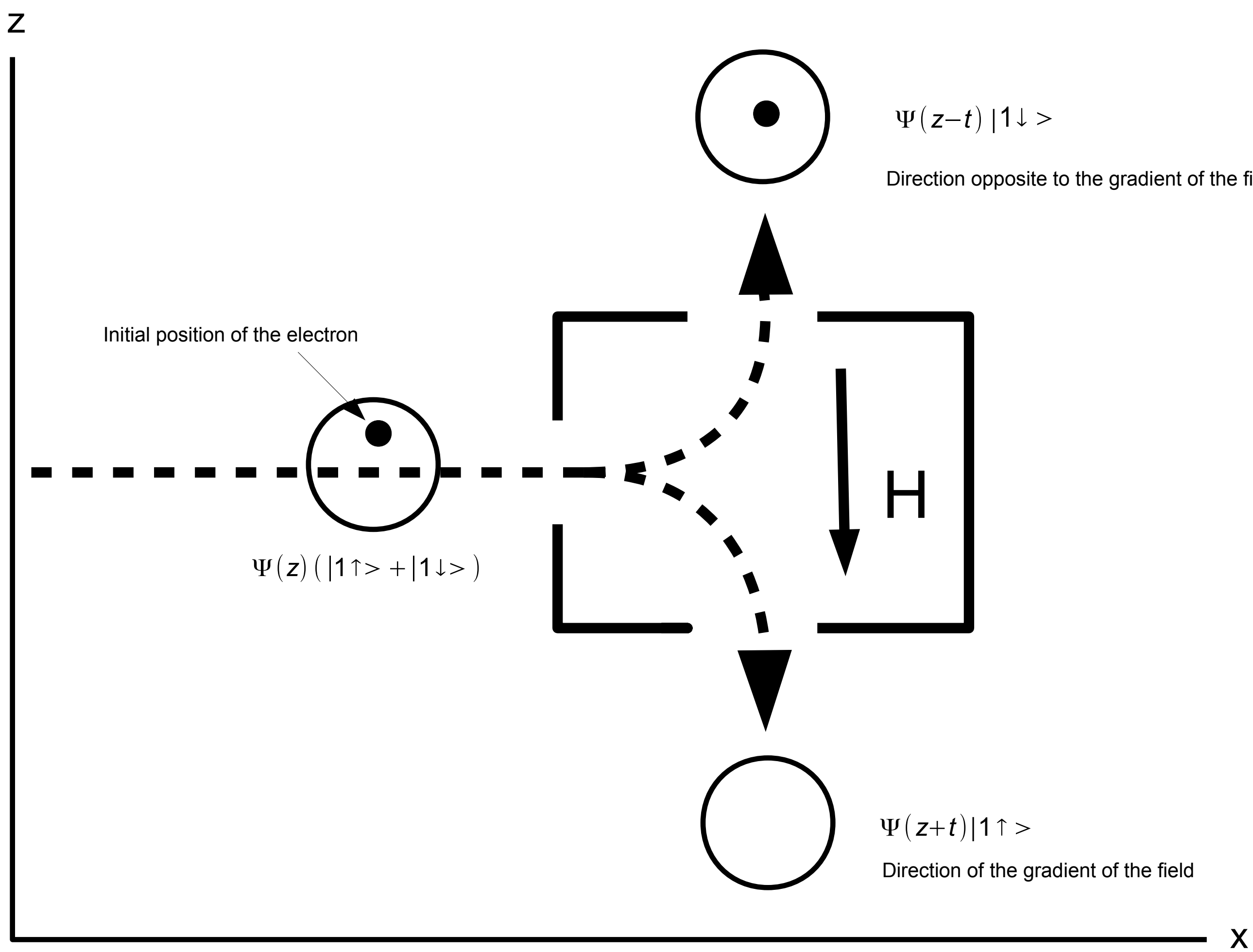
Now, repeat the same experiment, but with the direction of the gradient of the field reversed, and let us assume that the particle starts with *exactly the same wave function and the same position as before.*



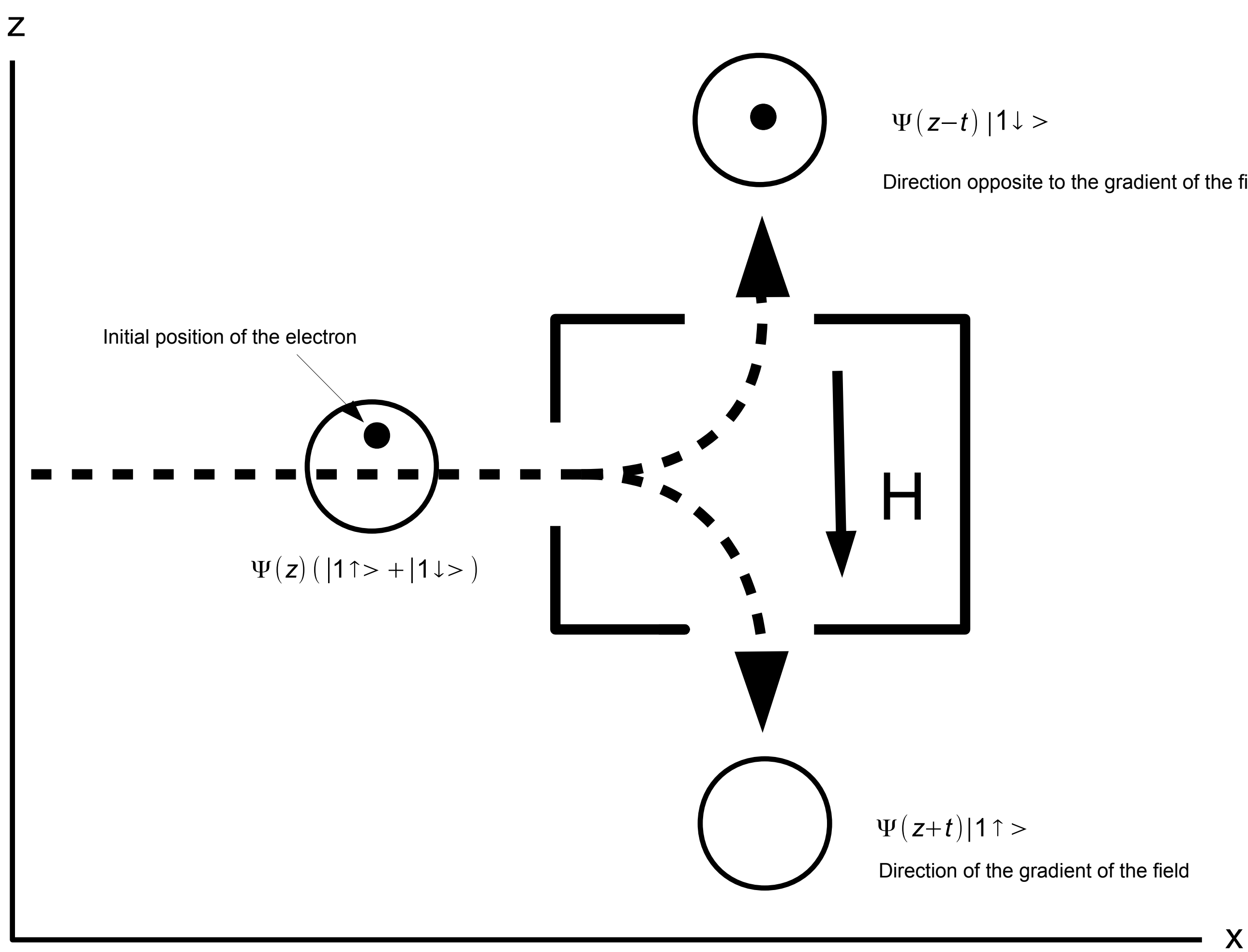
The particle is initially in the upper part of the support of the wave function, and, thus, it will still go upwards, because of the nodal line.



But going upwards means now going in the direction *opposite* to the one of the gradient of the field (since the latter is reversed).



So, the particle whose spin was "up", will "have" its spin "down", although one "measures" exactly the same quantity (the spin in the vertical direction), with *exactly the same initial conditions (for both the wave function and the position of the particle).*



So, with two different arrangements of the apparatus measuring the same spin operator, we get different results, for the same initial conditions of the particle.

WHAT ABOUT "MOMENTUM MEASUREMENTS"?

Consider a wave function

$$\Psi(x, 0) = \pi^{-1/4} \exp(-x^2/2),$$

and a set of particles whose density is given by

$$|\Psi(x, 0)|^2 = \pi^{-1/2} \exp(-x^2).$$

Since $\Psi(x, 0)$ is real, the phase S in $\Psi(x, 0) = R(x, 0)e^{iS(x,0)}$ vanishes and the guiding equation impies that all the particles are at rest:

$$\frac{d}{dt} X(t) = \frac{\partial S(X(t), t)}{\partial x} = 0.$$

Nevertheless the measurement of momentum p must have a probability density given by the square of the Fourier transform of

$$\Psi(x,0) = \pi^{-1/4} \exp(-x^2/2) :$$

$$|\hat{\Psi}(p)|^2 = \pi^{-1/2} \exp(-p^2).$$

Isn't that a contradiction?

But how does one **measure** that momentum? One way to do it is to let the particle move and to detect its asymptotic position $X(t)$ as $t \to \infty$. One has then $p = \lim_{t\to\infty} \frac{X(t)}{t}$ (with the mass $m = 1$).

Under a free evolution, the initial wave function $\Psi(x,0) = \pi^{-1/4}\exp(-x^2/2)$ will acquire an imaginary part.

That in turn will make the particle move, because of the guiding equation:

$$\frac{d}{dt}X(t) = \frac{\partial S(X(t),t)}{\partial x}$$

.

I may skip the details of the (easy) computation (see Appendix 1), but if one does it, one finds that, indeed, the variable $p = \lim_{t\to\infty} \frac{X(t)}{t}$ has the

$$|\hat{\Psi}(p)|^2 = \pi^{-1/2} \exp(-p^2)$$

distribution predicted by quantum mechanics, if the initial positon of the particle has the

$$|\Psi(x, 0)|^2 = \pi^{-1/2} \exp(-x^2)$$

distribution.

Therefore the apparatuses do not register something pre-existing to the "measurements", but play an active role.

When one "measures the spin", one does not find a pre-existing property of the particle alone (see Matthias lecture)!

When one "measures" the momentum, one finds a value which is not the instantaneous momentum.

So, in general (i.e. except for measurements of positions),

"MEASUREMENTS" DON'T MEASURE

AND "OBSERVATIONS" DON'T OBSERVE.)

Here is how Bell summarized the situation:

> "[...] the word [measurement] comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved."

Bell is here referring to statements of Bohr such as:

> "[. . .] the *impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.*"

The de Broglie-Bohm theory vindicates the intuition of Bohr and others about the role of the measuring device, but by making it a consequence of the theory and not some philosophical a priori.

It is often the orthodoxy that adopts a naive realism with respect to operators: the only thing we ever "see" are particles' positions, and the calculus with operators allows us to compute the statistics of those positions in certain types of interactions (called, misleadingly, measurements). But that does not mean that one has "measured" an operator (assuming that this expression makes sense).

WHAT ABOUT THE COLLAPSE OF THE WAVE FUNCTION IN THE DE BROGLIE-BOHM THEORY?

The wave function of the universe never collapses in the de Broglie-Bohm theory, but there is nevertheless an effective collapse.

In order to observe something, we need the particle to interact with a macroscopic system, because that is the only sort of thing we can perceive. A macroscopic system means that N is large, of the order of Avogadro's number $N \sim 10^{23}$. Such a system could be any detector in a laboratory, a pointer pointing up or down, a cat that can be alive or dead, etc.

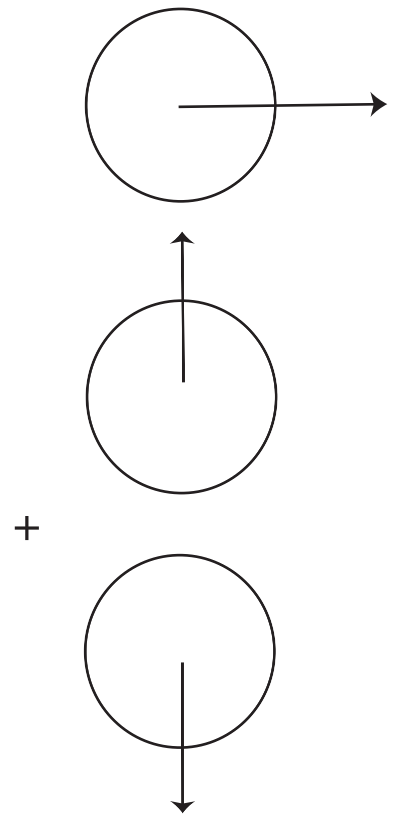
Consider a very simplified measurement process: the state

$$\Psi_0 = \varphi_0(z) \left[\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] ,$$

describes the original state of a particle whose spin is going to be measured, viz.,

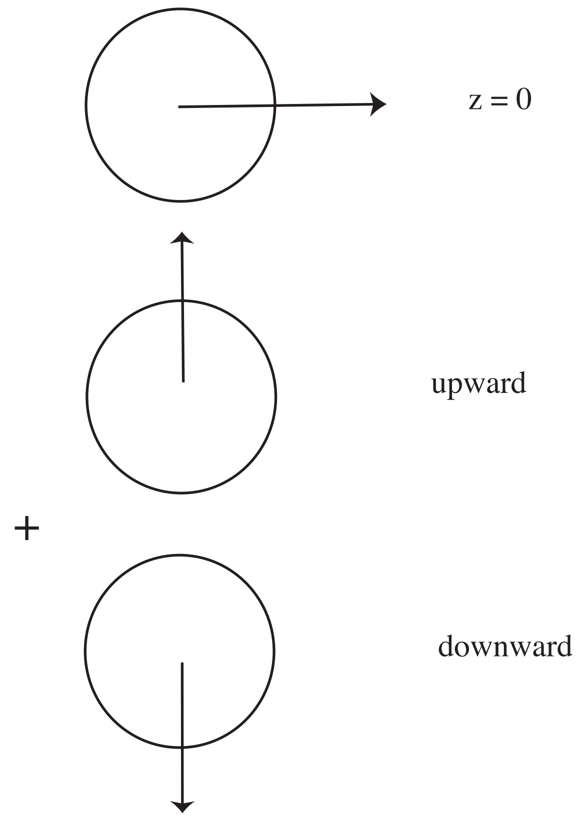
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} ,$$

and the state $\varphi_0(z)$ of the measuring device.



$$\Psi_0 = \varphi_0(z)\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\right] ,$$

Here z is a macroscopic variable, indicating the position of the measuring device (for example, the position of its center of mass along the vertical axis), and $\varphi_0(z)$ is centered at $z = 0$, i.e. the pointer is as in the first picture in the figure.

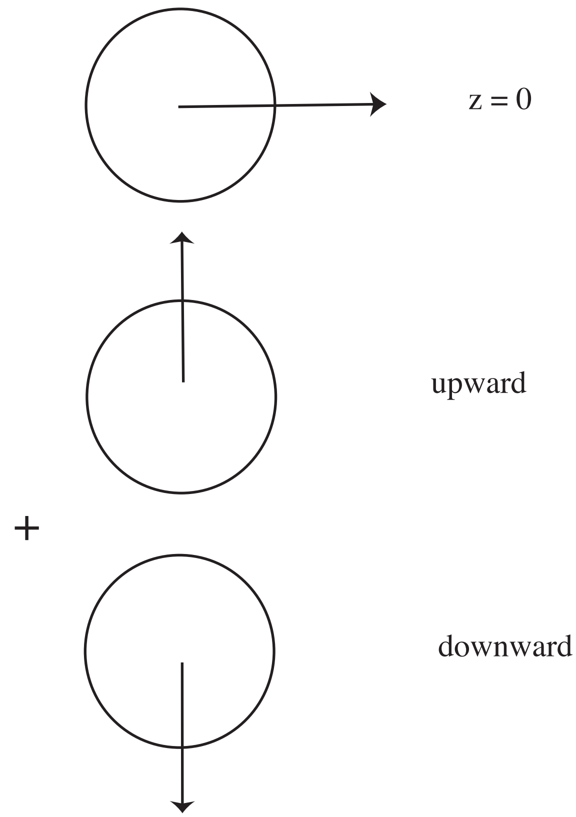


The state resulting after the measurement is

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\varphi^{\uparrow}(z)+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi^{\downarrow}(z)\ ,$$

where $\varphi^{\uparrow}(z)$ and $\varphi^{\downarrow}(z)$ correspond to the last two pictures in the figure, i.e., the pointer pointing upward or downward.

This follows immediately from the linearity of Schrödinger's equation.



Thus, the system is in a superposition of two macroscopically distinct states: one in which the pointer is pointing upward *and* one in which it is pointing downward. The problem is that we never see the pointer in such a superposed state: we see it *either* up *or* down, but not both.

But, in this example

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\varphi^{\uparrow}(z)+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi^{\downarrow}(z)\ ,$$

analyzed in the de Broglie-Bohm theory, the particle will be in the support of only one of the terms.

However, in principle, we must keep both terms because they may recombine later. But is such a recombination possible in practice for *macroscopic* systems? The answer is no, and the reason is that one would have to get the support of the quantum state to overlap again for *each* of the N variables.

And while this is possible for small N, it is quite a different matter to do it for $N \sim 10^{23}$. Thus, if we can be sure that no overlap will occur in the future between the two terms, we can simply keep the term in the support of which the particle happens to lie (and we know which one it is because of the coupling between the particle and the macroscopic device, by simply looking at the latter), as far as the predictions for the future behavior of the system are concerned.

It is sometimes thought that, for such an effective collapse to occur, one needs the measuring device to interact with an environment, such as the air molecules surrounding it, and ultimately the entire universe. But that is not true: any sufficiently macroscopic device suffices, even if the latter were perfectly isolated from the rest of the universe (which is never the case, but that is not relevant since what we say would be true even if perfect isolation were possible).

So, in some sense, we do “collapse” the quantum state when we look at the result of an experiment. But this is only a practical matter. We can still consider that the true quantum state is and remains forever given by the time evolution of the full quantum state

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\varphi^{\uparrow}(z)+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi^{\downarrow}(z)\ ,$$

It is simply that one of the terms of the quantum state no longer guides the motion of the particle, either now or at any time in the future.

The measuring process here is an entirely physical process, with no role whatsoever left to the observer.

This effect, namely the fact that the two terms in

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\varphi^{\uparrow}(z)+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi^{\downarrow}(z)\ ,$$

will not overlap or interfere in the future, is called *decoherence* and has been the subject of an extensive literature. But, for our purposes, the basic idea, already given by David Bohm in 1952 and outlined here, is sufficient.

Some people think that decoherence alone solves the measurement problem (see Matthias's lecture), the argument being that, since the two terms in

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}\varphi^{\uparrow}(z)+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi^{\downarrow}(z)\ ,$$

do not overlap or interfere in the future, we just pick up the one we see in order to predict the future behavior of the particle.

But that puts back the observer on center stage since there is no fact of the matter distinguishing the two terms except **what we observe**, unlike what happens in the de Broglie-Bohm theory, where the difference between the two terms is that the particle is in the support of only one of them.

SUMMARY:

The main virtue of the de Broglie–Bohm theory is that it is a clear theory about what is going on in the world, whether we look at it or not. So the vagueness and subjectivity of the notion of "observer" or of "measurement" simply disappear in this theory.

The de Broglie–Bohm theory and ordinary quantum mechanics are *not* the same theory (although they make the same predictions), because the de Broglie–Bohm theory *is* a theory about microscopic reality outside the laboratories, while ordinary quantum mechanics *is not*: it is an algorithm for very accurately predicting results of experiments, an algorithm that is, in fact, a consequence of the de Broglie–Bohm theory.

The de Broglie–Bohm theory is simply the rational completion of ordinary quantum mechanics!

To quote Bell again:

> “Why this necessity to refer to ‘apparatus’ when we would discuss quantum phenomena? The physicists who first came upon such phenomena found them so bizarre that they despaired of describing them in terms of ordinary concepts like space and time, position and velocity. The founding fathers of quantum theory decided even that no concepts could possibly be found which could permit direct description of the quantum world. So the theory which they established aimed only to describe systematically the response of the apparatus. And what more, after all, is needed for applications? [...]”

“The ‘Problem’ then is this: how exactly is the world to be divided into a speakable apparatus … that we can talk about … and unspeakable quantum system that we can not talk about? How many electrons, or atoms, or molecules, make an ‘apparatus’? The mathematics of the ordinary theory requires such a division, but says nothing about how it is to be made. In practice the question is resolved by pragmatic recipes which have stood the test of time. But should not fundamental theory permit exact mathematical formulation?”

“Now in my opinion the founding fathers were in fact wrong on this point. The quantum phenomena do *not* exclude a uniform description of micro and macro worlds . . . system and apparatus. It is *not* essential to introduce a vague division of the world of this kind. This was indicated already in 1926 by de Broglie, when he answered the conundrum

wave or particle ?

by

wave *and* particle.”

"But by the time this was fully clarified by Bohm in 1952, few theoretical physicists wanted to hear about it. The orthodox line seemed fully justified by practical success. Even now the de Broglie-Bohm picture is generally ignored, and not taught to students. I think this is a great loss. For that picture exercises the mind in a very salutary way.

The de Broglie-Bohm picture disposes of the necessity to divide the world somehow into system and apparatus."

John Bell

John Bell also recalled the arguments claiming to show that a theory such as the de Broglie–Bohm one is impossible and added:

> "But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one."

“More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the ‘observer’, could be eliminated.”

John Bell

Bell was also wondering:

> "Why is the pilot wave picture ignored in textbooks? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced on us by experimental facts, but by deliberate theoretical choice?"

John Bell

Let me end by quoting an ex-physics student, whose sentiments are close to mine when I was a student:

> "My interest has always been to understand what the world is like. This is the main reason that I majored in physics: if physics is the study of nature, then to understand nature one should learn physics first. But my hopes were disappointed by what is (or at least seems to be) commonly accepted in many physics departments all over the world: after quantum mechanics, we should give up the idea that physics provides us with a picture of reality."

“At first, I believed this was really the case and I was so disappointed that I decided to forget about my “romantic” dream. At some point, [...] I realized that some of the things I took for granted were not so obviously true, and I started to regain hope that quantum mechanics was not really the “end of physics” as I meant it.”

"Therefore, I decided to go to graduate school in physics to figure out what the situation really was. While taking my PhD in the foundations of quantum mechanics, I understood that what physicists thought was an unavoidable truth was instead a blunt mistake: quantum mechanics does not force us to give up anything, and certainly not the possibility to investigate reality through physics."

Valia Allori

APPENDIX 1: COMPUTATION OF THE MOMENTUM DISTRIBUTION.

The solution of Schrödinger's equation,

$$i\frac{\partial\Psi(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial^2\Psi(x,t)}{\partial x^2}$$

with initial wave function

$$\Psi(x,0) = \pi^{-1/4}\exp(-x^2/2),$$

is:

$$\Psi(x,t) = \frac{1}{(1+it)^{1/2}}\frac{1}{\pi^{1/4}}\exp\left[-\frac{x^2}{2(1+it)}\right] .$$

So,

$$\Psi(x,t) = \frac{1}{(1+it)^{1/2}}\frac{1}{\pi^{1/4}}\exp\left[-\frac{x^2}{2(1+it)}\right] .$$

If one writes $\Psi(x,t) = R(x,t)\exp\left[iS(x,t)\right]$, one has (up to a constant in x):

$$S(x,t) = \frac{tx^2}{2(1+t^2)} ,$$

and the guiding equation ($\frac{d}{dt}X(t) = \frac{\partial S(X(t),t)}{\partial x}$) becomes:

$$\frac{d}{dt}X(t) = \frac{tX(t)}{1+t^2} ,$$

whose solution is:

$$X(t) = X(0)\sqrt{1+t^2}$$

$$X(t) = X(0)\sqrt{1+t^2}$$

gives the explicit dependence of the position of the particle as a function of time. If the particle is initially at $X(0) = 0$, it does not move; otherwise, it moves asymptotically, when $t \to \infty$, as $X(t) \sim X(0)t$.

Thus,

$$p = \lim_{t \to \infty} \frac{X(t)}{t} = \lim_{t \to \infty} \frac{X(0)\sqrt{1+t^2}}{t} = X(0),$$

Since we started with

$$\Psi(x,0) = \pi^{-1/4} \exp(-x^2/2),$$

hence, a density distribution of $X(0) = x$:

$$|\Psi(x,0)|^2 = \pi^{-1/2} \exp(-x^2),$$

we get that the density distribution for p is given by:

$$\pi^{-1/2} \exp(-p^2) = |\hat{\Psi}(p,0)|^2,$$

which is the quantum prediction!

But this does not measure the instantaneous momentum (equal to zero).

References:

(1) Albert D., *Quantum Mechanics and Experience.* A pedagogical and elementary Introduction to the quantum formalism and various "interpretations" : the de Broglie-Bohm theory, spontaneous collapse and many-worlds.

(2) Bell J. S., *Speakable and Unspeakable in Quantum Mechanics. Collected Papers on Quantum Philosophy.*

(3) Bohm D., Hiley B.J., *The Undivided universe.* Bohm's viewpoint.

(4) Bricmont J. *Making Sense of Quantum Mechanics.*

(5) Bricmont J. *Quantum Sense and Nonsense* A more "popular" book.

(6) Dürr D., Teufel S., *Bohmian Mechanics. The Physics and Mathematics of Quantum Theory.* Rigorous mathematical treatment of the de Broglie-Bohm theory.

(7) D. Dürr, S. Goldstein and N. Zanghì: *Quantum Physics Without Quantum Philosophy*, collection of their articles.

(8) Holland P., *The Quantum Theory of Motion. An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics.* A more physics-oriented approach to the de Broglie-Bohm theory.

(9) Maudlin, T. *Philosophy of Physics: Quantum Theory.* Analysis of the de Broglie-Bohm theory, spontaneous collapse and many-worlds by one of the best contemporary philosophers of science.

(10) Norsen, T., *Foundations of Quantum Mechanics. An Exploration of the Physical Meaning of Quantum Theory.* A relatively elementary and pedagogical discussion of various "interpretations" of quantum mechanics: the de Broglie-Bohm theory, spontaneous collapse and many-worlds, from a viewpoint favorable to de Broglie-Bohm theory.

(11) R. Tumulka: Understanding Bohmian mechanics — A dialogue, *American Journal of Physics* **72**, 1220–1226 (2004)

(12) Tumulka, R. Lecture Notes on Foundations of Quantum Mechanics https://www.math.uni-tuebingen.de/de/forschung/maphy/personen/roderi
ws2017-18/quantmech/dateien/script2017.pdf