Collapse Theories

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- “Collapse theories” means theories of spontaneous collapse of the wave function, i.e., collapse not triggered by “measurements” done by “observers.”
- The simplest such theory is due to Ghirardi, Rimini, and Weber (GRW).
- People who also published on this approach include Philip Pearle, Angelo Bassi, Nicolas Gisin, Roger Penrose, Antony Leggett, and Steven Weinberg.

Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger's cat) of macroscopically different contributions.

Put differently, regard the collapse of ψ as a physical process governed by mathematical laws.

GianCarlo Ghirardi (1935–2018)

Explicit equations by Ghirardi, Rimini, and Weber [Phys.Rev. D 1986]

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.

GRW's stochastic evolution for ψ

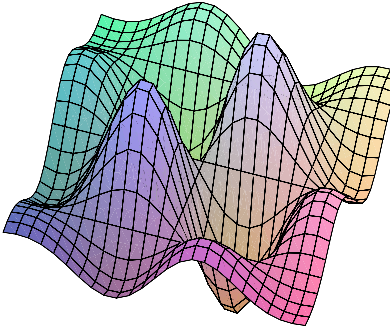
- is designed for non-relativistic quantum mechanics of N particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-16}\,\mathrm{sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7}\,\mathrm{m}$, called collapse width.
- Def: ψ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy σ.
- "rate $N\lambda$" means that
  prob(an event in the next dt seconds) $= N\lambda\, dt$.
- more explicitly: Schrödinger evolution interrupted by jumps of the form

  $$\psi_{T+} = e^{-\frac{(\boldsymbol{q}_k-\boldsymbol{q})^2}{4\sigma^2}}\,\psi_{T-}\,,$$

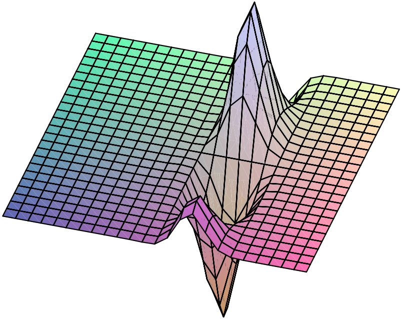
  i.e., multiplication by a Gauss function with random label k, center $\boldsymbol{q}$ and time T.
- prob$(\boldsymbol{q} \in d^3\boldsymbol{q}) = \|\psi_{T+}\|^2 d^3\boldsymbol{q} = |\psi_{T-}(\boldsymbol{q}_k = \boldsymbol{q})|^2 * \text{Gaussian}$

GRW's spontaneous collapse

before the "spontaneous collapse" :



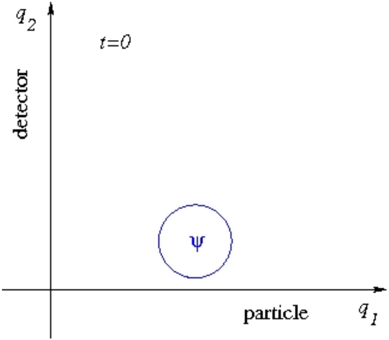
and after:



- In Hilbert space: piecewise deterministic stochastic jump process. ψ_t jumps at random times to random destinations.
- For a single particle, one collapse every 100 million years.
- For 10^4 particles, one collapse every 10,000 years.
- For 10^{23} particles, one collapse every 10^{-7} seconds.
- No-signaling theorem

How GRW theory solves the measurement problem

Evolution of ψ in configuration space of particle + detector:



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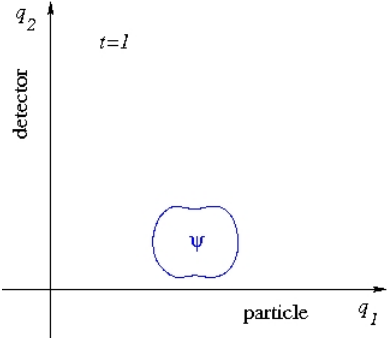

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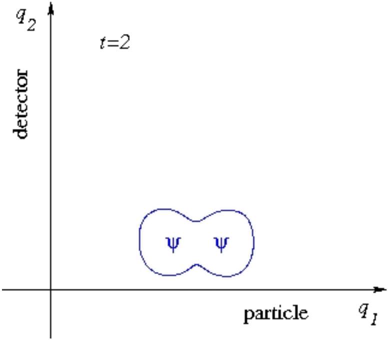

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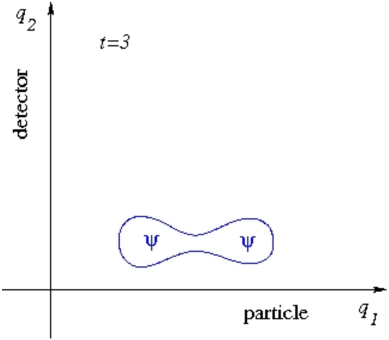

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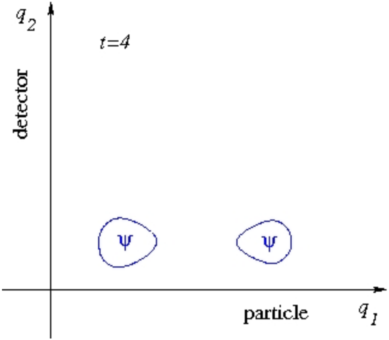

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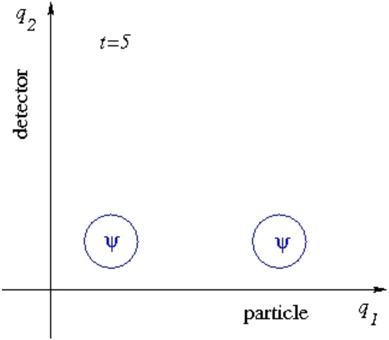

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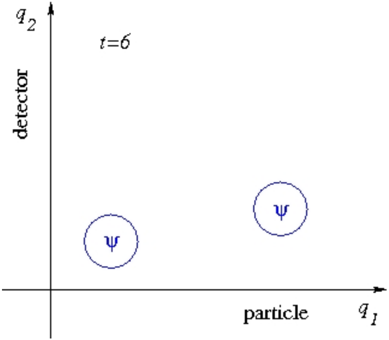

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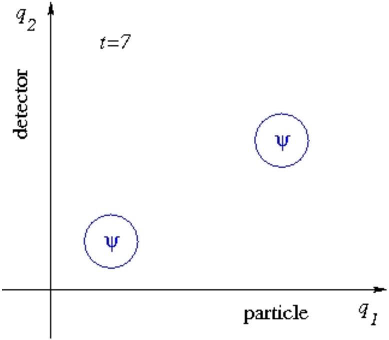

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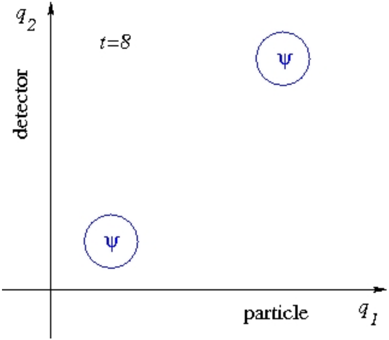

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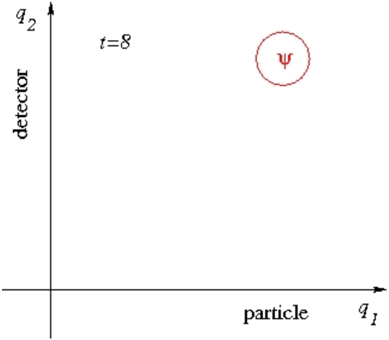
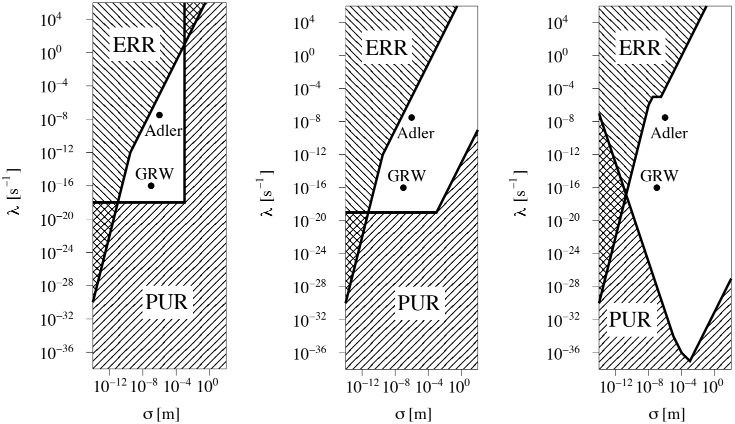


- As soon as a collapse occurs for one particle in the apparatus, the superposition in the test particle is gone as well.
- A macroscopic superposition $\sum_i \psi_i$ such as Schrödinger's cat would collapse within 10^{-7} seconds.
- It would collapse, up to tails of the Gaussian, to one of the macroscopically distinct wave packets ψ_i (to either $|\text{dead}\rangle$ or $|\text{alive}\rangle$).
- The probability that ψ collapses to ψ_i is, up to Gaussian tails, given by $\|\psi_i\|^2$.
- That is why GRW theory agrees with the standard quantum prediction to an excellent degree of approximation.

- But in principle, the predictions of GRW theory can differ from standard QM.
- For example, in a double slit experiment in which it takes the particle 300 million years to travel from the double slit to the screen, the interference pattern would disappear.
- It is not easy to test GRW against standard QM.
- Dramatic energy increase for much smaller σ values than 10^{-7} m (exercise)
- Slight energy increase for $\sigma = 10^{-7}$ m (exercise).

GRW theories are empirically adequate

Their predictions deviate very very slightly from the quantum formalism.



GRWf GRWm CSLm

Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region [Feldmann, Tumulka arXiv:1109.6579]

- How can non-commuting operators as observables arise in a contradiction-free theory?
- Wasn't non-commutativity proof of complementarity?
- But we have seen in the measurement problem how GRW theory reproduces the predictions of standard QM.
- In GRW theory, the distribution of the collapse center $\boldsymbol{q}$ has density $\|\psi_{T+}\|^2 = \|e^{-(\boldsymbol{q}_k-\boldsymbol{q})^2/4\sigma^2}e^{-iHt}\psi_0\|^2 =$ $\langle\psi_0|e^{iHt}e^{-(\boldsymbol{q}_k-\boldsymbol{q})^2/2\sigma^2}e^{-iHt}|\psi_0\rangle = \langle\psi_0|G(T,\boldsymbol{q})|\psi_0\rangle$, and the $G(T,\boldsymbol{q})$ operators do not generally commute for different values of T.

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It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.

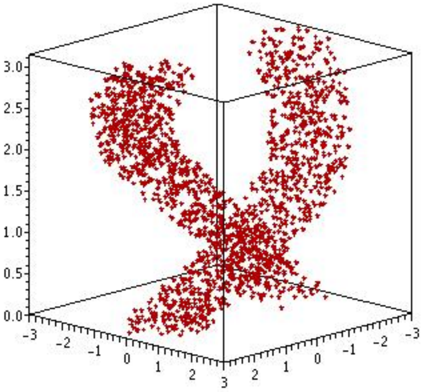
The word “beables”

- Bell coined the word as be-ables in contrast to observ-ables.
- Beables are the variables that represent the ontology (the things that exist), the quantities that actually have values (in contrast to observables).
- The word “beables” is also meant to suggest a tentative character (meaning “could be”) because what the beables are depends on the theory. Different theories have different pictures of what is real in the world.

The expression "primitive ontology"

- Def: Primitive ontology is the part of the ontology that represents matter in 3d space (or 4d space-time).
- Example: In Bohmian mechanics, the ontology consists of the particles and the wave function; the primitive ontology consists of the particles.
- I think that for GRW theory to make sense, it needs a primitive ontology.
- Here are two proposals: "flash ontology" and "matter density ontology."

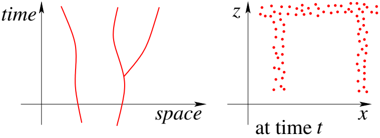
Flash ontology



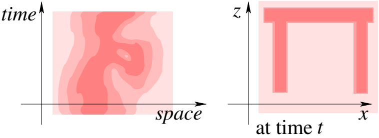
Instead of particle world lines, there are world points in space-time, called "flashes." A macroscopic object consists of a galaxy of flashes.

Ontologies

- Suppose a theory T talks a about particles in the literal sense, having world lines in space-time.



- Then we say that T has a particle ontology.
- Examples: Classical mechanics, Bohmian mechanics.
- Now suppose that a theory T' says that matter is continuously distributed in 4D space-time, with density function $m(t,\boldsymbol{x})$ [or $m_\mu(t,\boldsymbol{x})$ or $m_{\mu\nu}(t,\boldsymbol{x})$].



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Laws for the primitive ontology

Def: GRWf [Bell 1987]

If ψ collapses at time T with center $\boldsymbol{q}$ then put a flash at $(T, \boldsymbol{q})$.

Def: GRWm [Diósi 1989; Ghirardi, Grassi, Benatti 1995; Goldstein 1998]

matter is continuously distributed with density given by

$$\begin{aligned} m(t,\boldsymbol{q}) &= \sum_{k=1}^{N} m_k \int \delta^3(\boldsymbol{q}-\boldsymbol{q}_k)\, |\psi_t(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_N)|^2\, d^3\boldsymbol{q}_1 \cdots d^3\boldsymbol{q}_N \\ &= \langle \psi_t | \mathcal{M}(\boldsymbol{x}) | \psi_t \rangle \end{aligned}$$

with $\mathcal{M}(\boldsymbol{x}) = \sum_{k=1}^{N} m_k\, \delta^3(\boldsymbol{x}-\hat{\boldsymbol{Q}}_k)$ the mass density operators.

GRWf and GRWm are empirically equivalent (exercise).

Why we need a primitive ontology (1)

- There is a logical gap between saying

$$\text{“}\psi\text{ is the wave function of a live cat”} \tag{1}$$

  and saying

$$\text{“there is a live cat.”} \tag{2}$$

- After all, in Bohmian mechanics, (2) follows from (1) by virtue of a law of the theory, $Q_t \sim |\psi_t|^2$.
- Imagine Bohmian particles guided by a GRW wave function [Allori et al. arXiv:1206.0019]. The particles behave in a catastrophic way, although the wave function looks reasonable. So if you haven't specified the primitive ontology, you don't know what cats or pointers do.

Why we need a primitive ontology (2)

- Without it, paradoxes arise:
- **Paradox:** One might think GRW fails to solve the measurement problem: suppose

  $$\psi = c_1\,\psi_1 + c_2\psi_2$$

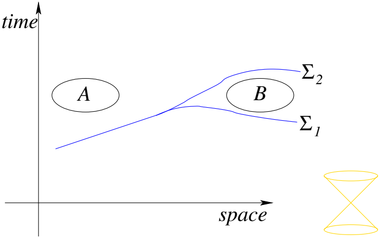
  is a superposition of macroscopically different states ψ_1, ψ_2. If $c_1 = \sqrt{0.5} = c_2$ then there is a problem; if $c_1 = \sqrt{0.4}$ and $c_2 = \sqrt{0.6}$ then there is still a problem. How small would c_1 have to be for the problem to disappear?
- **Answer:** The reasoning misses the primitive ontology. In GRWm, e.g., an *m* function close to that of a live cat is still an *m* function of a live cat.

Why we need a primitive ontology (3)

- Another one:
- **Paradox:** Suppose $|c_1|^2$ is near 1 and $|c_2|^2$ near 0. If we made a measurement of the macrostate, there is a positive probability $|c_2|^2$ for finding ψ_2. So how can we say that a cat with $\psi = c_1|\text{dead}\rangle + c_2|\text{alive}\rangle$ is really dead?
- **Answer:** In GRWm, "the cat is dead" means that the *m* function looks and behaves like a dead cat. The measurement might change ψ to near $|\text{alive}\rangle$, and then the cat *is* alive in GRWm. So, GRWm allows for resurrections—with tiny probability!

Why we need a primitive ontology (4)

A problem about relativistic facts for GRW0:



Consider an EPR experiment, in which two particles in the singlet spin state are widely separated in space, and a Stern–Gerlach experiment is carried out on each particle. The reduced spin state ρ of particle A (obtained by tracing out the spin of particle B) will depend on the choice of hypersurface Σ: If $\Sigma = \Sigma_2$ lies after the experiment on particle B but before that on particle A, then ρ will be a pure state. If $\Sigma = \Sigma_1$ lies before both experiments, ρ will be mixed.

This poses a problem of finding a consistent relativistic specification of facts for GRW0. However, the problem evaporates for GRWf/m.

Limitations to knowledge in GRW theories

A "quantum measurement" is not a measurement in the literal sense (discovering a pre-existing value). (See Jean's lecture on no-hidden-variables theorems.)

How then about measurements in the literal sense: Can we measure, e.g., the number of collapses in a system (e.g., water droplet with 10^{15} molecules) during the time interval $[t_1, t_2]$? Is there a "Geiger counter for collapses"?

Short answer: no

There are limitations to knowledge: Nature knows the exact number of collapses, but inhabitants can find it out only with macroscopic inaccuracy (say, $\pm 10^9$ per second).

[Cowan and Tumulka arXiv:1307.0810, 1307.0827, 1312.7321]

What is good and what is bad about collapse theories (1)

Some people say:

- GRW is better than Bohm or many-worlds becaue you can test it against standard QM.
  I don't see why that would be a reason to prefer a theory.
- GRW is good because then the ontology is "ψ only."
  I don't think the theory is satisfactory with ψ-only ontology (GRW$\emptyset$).

What is good and what is bad about collapse theories (2)

What is good:

- In relativistic space-time, Bohmian mechanics requires a preferred foliation, which seems against the spirit of relativity. I think the possibility of a preferred foliation should be taken seriously. But if this is your main concern about Bohmian mechanics, you should like GRW b/c it doesn't require a preferred foliation. (See Matthias' lecture on relativity)
- It is less radical than many-worlds, and free of the problem of counting worlds.

What is bad:

- "It seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist." (L. de Broglie 1927)
- Both Bohmian mechanics and many-worlds are so much, much simpler, both mathematically (no stochastic processes) and conceptually.
- Bohm's ontology (particles) is more natural.
- GRW theory is motivated by the "ψ-only" idea, but this idea doesn't work in the end.

One more thing that is good about GRW theory:

- The big divide in the literature about how to understand quantum theory is between proposals that provide a coherent picture (Bohm, GRW, many-worlds) and those that don't (Copenhagen, decoherent histories).
  GRW theory provides what Bohr claimed was impossible: a contradiction-free, empirically adequate explanation of our world.

Thank you for your attention