Collapse Theories

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“Collapse theories” means theories of spontaneous collapse of the wave function, i.e., collapse not triggered by “measurements” done by “observers.”

The simplest such theory is due to Ghirardi, Rimini, and Weber (GRW).

People who also published on this approach include Philip Pearle, Angelo Bassi, Nicolas Gisin, Roger Penrose, Antony Leggett, and Steven Weinberg.
Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles \(N < 10^4\) but not for macroscopic systems \(N > 10^{23}\). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger’s cat) of macroscopically different contributions. Put differently, regard the collapse of \(\psi\) as a physical process governed by mathematical laws.


The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.
GRW’s stochastic evolution for $\psi$

- is designed for non-relativistic quantum mechanics of $N$ particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-16} \text{sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7} \text{m}$, called collapse width.
- Def: $\psi$ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy $\sigma$.
- “rate $N\lambda$” means that
  $\text{prob( an event in the next } dt \text{ seconds) } = N\lambda \ d t$.
- more explicitly: Schrödinger evolution interrupted by jumps of the form
  \[ \psi_{T+} = e^{-\frac{(q_k - q)^2}{4\sigma^2}} \psi_{T-}, \]
  i.e., multiplication by a Gauss function with random label $k$, center $q$ and time $T$.
- $\text{prob}(q \in d^3q) = \|\psi_{T+}\|^2 d^3q = |\psi_{T-}(q_k = q)|^2 * \text{Gaussian}$
before the “spontaneous collapse”:

and after:
In Hilbert space: piecewise deterministic stochastic jump process. $\psi_t$ jumps at random times to random destinations.

- For a single particle, one collapse every 100 million years.
- For $10^4$ particles, one collapse every 10,000 years.
- For $10^{23}$ particles, one collapse every $10^{-7}$ seconds.
- No-signaling theorem
How GRW theory solves the measurement problem
Evolution of $\psi$ in configuration space of particle + detector:

$t=0$

Detector $q_2$

Particle $q_1$
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$t = 5$

[Diagram showing two positions of $\psi$, one for the particle and one for the detector.]
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As soon as a collapse occurs for one particle in the apparatus, the superposition in the test particle is gone as well.

A macroscopic superposition $\sum_i \psi_i$ such as Schrödinger’s cat would collapse within $10^{-7}$ seconds.

It would collapse, up to tails of the Gaussian, to one of the macroscopically distinct wave packets $\psi_i$ (to either $|\text{dead}\rangle$ or $|\text{alive}\rangle$).

The probability that $\psi$ collapses to $\psi_i$ is, up to Gaussian tails, given by $||\psi_i||^2$.

That is why GRW theory agrees with the standard quantum prediction to an excellent degree of approximation.
But in principle, the predictions of GRW theory can differ from standard QM.

For example, in a double slit experiment in which it takes the particle 300 million years to travel from the double slit to the screen, the interference pattern would disappear.

It is not easy to test GRW against standard QM.

Dramatic energy increase for much smaller $\sigma$ values than $10^{-7}$ m (exercise)

Slight energy increase for $\sigma = 10^{-7}$ m (exercise).
GRW theories are empirically adequate

Their predictions deviate very very slightly from the quantum formalism.

How can non-commuting operators as observables arise in a contradiction-free theory?

Wasn’t non-commutativity proof of complementarity?

But we have seen in the measurement problem how GRW theory reproduces the predictions of standard QM.

In GRW theory, the distribution of the collapse center $q$ has density

$$\|\psi_{T+}\|^2 = \|e^{-\frac{(q_k-q)^2}{4\sigma^2}} e^{-iHt} \psi_0\|^2 = \langle \psi_0 | e^{iHt} e^{-(q_k-q)^2/2\sigma^2} e^{-iHt} | \psi_0 \rangle = \langle \psi_0 | G(T, q) | \psi_0 \rangle,$$

and the $G(T, q)$ operators do not generally commute for different values of $T$. 

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It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.
Bell coined the word as be-ables in contrast to observ-ables.

Beables are the variables that represent the ontology (the things that exist), the quantities that actually have values (in contrast to observables).

The word “beables” is also meant to suggest a tentative character (meaning “could be”) because what the beables are depends on the theory. Different theories have different pictures of what is real in the world.
Def: Primitive ontology is the part of the ontology that represents matter in 3d space (or 4d space-time).

Example: In Bohmian mechanics, the ontology consists of the particles and the wave function; the primitive ontology consists of the particles.

I think that for GRW theory to make sense, it needs a primitive ontology.

Here are two proposals: “flash ontology” and “matter density ontology.”
Instead of particle world lines, there are world points in space-time, called “flashes.” A macroscopic object consists of a galaxy of flashes.
Suppose a theory $T$ talks about particles in the literal sense, having world lines in space-time.

Then we say that $T$ has a particle ontology.

Examples: Classical mechanics, Bohmian mechanics.

Now suppose that a theory $T'$ says that matter is continuously distributed in 4D space-time, with density function $m(t, x)$ [or $m_\mu(t, x)$ or $m_{\mu\nu}(t, x)$].

Then we say that $T'$ has a matter density ontology.
Laws for the primitive ontology

**Def: GRWf**

[Bell 1987]

If $\psi$ collapses at time $T$ with center $q$ then put a flash at $(T, q)$.

**Def: GRWm**

[Diósi 1989; Ghirardi, Grassi, Benatti 1995; Goldstein 1998]

Matter is continuously distributed with density given by

$$m(t, q) = \sum_{k=1}^{N} m_k \int \delta^3(q - q_k) |\psi_t(q_1, \ldots, q_N)|^2 \ d^3q_1 \cdots d^3q_N$$

$$= \langle \psi_t | M(x) | \psi_t \rangle$$

with $M(x) = \sum_{k=1}^{N} m_k \delta^3(x - \hat{Q}_k)$ the mass density operators.

GRWf and GRWm are empirically equivalent (exercise).
There is a logical gap between saying

\[ \psi \text{ is the wave function of a live cat} \]  

and saying

\[ \text{there is a live cat.} \]  

After all, in Bohmian mechanics, (2) follows from (1) by virtue of a law of the theory, \( Q_t \sim |\psi_t|^2 \).

Imagine Bohmian particles guided by a GRW wave function [Allori et al. arXiv:1206.0019]. The particles behave in a catastrophic way, although the wave function looks reasonable. So if you haven’t specified the primitive ontology, you don’t know what cats or pointers do.
Without it, paradoxes arise:

**Paradox:** One might think GRW fails to solve the measurement problem: suppose

$$\psi = c_1 \psi_1 + c_2 \psi_2$$

is a superposition of macroscopically different states $\psi_1, \psi_2$. If $c_1 = \sqrt{0.5} = c_2$ then there is a problem; if $c_1 = \sqrt{0.4}$ and $c_2 = \sqrt{0.6}$ then there is still a problem. How small would $c_1$ have to be for the problem to disappear?

**Answer:** The reasoning misses the primitive ontology. In GRWm, e.g., an $m$ function close to that of a live cat is still an $m$ function of a live cat.
Another one:

**Paradox:** Suppose $|c_1|^2$ is near 1 and $|c_2|^2$ near 0. If we made a measurement of the macrostate, there is a positive probability $|c_2|^2$ for finding $\psi_2$. So how can we say that a cat with $\psi = c_1|\text{dead}\rangle + c_2|\text{alive}\rangle$ is really dead?

**Answer:** In GRWm, “the cat is dead” means that the $m$ function looks and behaves like a dead cat. The measurement might change $\psi$ to near $|\text{alive}\rangle$, and then the cat *is* alive in GRWm. So, GRWm allows for resurrections—with tiny probability!
Consider an EPR experiment, in which two particles in the singlet spin state are widely separated in space, and a Stern–Gerlach experiment is carried out on each particle. The reduced spin state ρ of particle A (obtained by tracing out the spin of particle B) will depend on the choice of hypersurface Σ: If Σ = Σ₂ lies after the experiment on particle B but before that on particle A, then ρ will be a pure state. If Σ = Σ₁ lies before both experiments, ρ will be mixed.

This poses a problem of finding a consistent relativistic specification of facts for GRW₀. However, the problem evaporates for GRWf/m.
A “quantum measurement” is not a measurement in the literal sense (discovering a pre-existing value). (See Jean’s lecture on no-hidden-variables theorems.)

How then about measurements in the literal sense: Can we measure, e.g., the number of collapses in a system (e.g., water droplet with $10^{15}$ molecules) during the time interval $[t_1, t_2]$? Is there a “Geiger counter for collapses”?

Short answer: no

There are limitations to knowledge: Nature knows the exact number of collapses, but inhabitants can find it out only with macroscopic inaccuracy (say, $\pm 10^9$ per second).

[Cowan and Tumulka arXiv:1307.0810, 1307.0827, 1312.7321]
Some people say:

- GRW is better than Bohm or many-worlds because you can test it against standard QM.
  I don’t see why that would be a reason to prefer a theory.
- GRW is good because then the ontology is “ψ only.”
  I don’t think the theory is satisfactory with ψ-only ontology (GRWØ).
What is good:

- In relativistic space-time, Bohmian mechanics requires a preferred foliation, which seems against the spirit of relativity. I think the possibility of a preferred foliation should be taken seriously. But if this is your main concern about Bohmian mechanics, you should like GRW b/c it doesn’t require a preferred foliation. (See Matthias’ lecture on relativity)
- It is less radical than many-worlds, and free of the problem of counting worlds.

What is bad:

- “It seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist.” (L. de Broglie 1927)
- Both Bohmian mechanics and many-worlds are so much, much simpler, both mathematically (no stochastic processes) and conceptually.
- Bohm’s ontology (particles) is more natural.
- GRW theory is motivated by the “ψ-only” idea, but this idea doesn’t work in the end.
One more thing that is good about GRW theory:

- The big divide in the literature about how to understand quantum theory is between proposals that provide a coherent picture (Bohm, GRW, many-worlds) and those that don’t (Copenhagen, decoherent histories).
- GRW theory provides what Bohr claimed was impossible: a contradiction-free, empirically adequate explanation of our world.
Thank you for your attention