

Vektoren und Matrizen

Vektor = n -Tupel

$$u = (u_1, \dots, u_n) = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

Cartesisches Produkt

$$A \times B = \{ (a, b) : a \in A, b \in B \}$$

$$A_1 \times \dots \times A_n = \{ (a_1, \dots, a_n) : a_1 \in A_1, \\ a_2 \in A_2, \dots, a_n \in A_n \}$$

$$\underbrace{A \times A \times \dots \times A}_{n \text{ Faktoren}} =: A^n, \quad \mathbb{R}^3$$

Matrix A = Schema mit $m \cdot n$ Einträgen.

$$\text{z.B. } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (m=2=n)$$

$$\text{allg. } A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{m,n} \end{bmatrix} \quad \begin{array}{l} m \text{ Zeilen} \\ n \text{ Spalten} \end{array}$$
$$= (a_{ij})$$

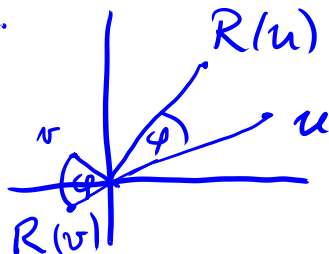
Anwendung einer Matrix auf einen Vektor: $\in \mathbb{R}^2$

$$\underline{\text{Def}} \quad Au := \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} := \begin{bmatrix} a_{11}u_1 + a_{12}u_2 \\ a_{21}u_1 + a_{22}u_2 \end{bmatrix}$$

Matrix \cdot Vektor = Vektor.

Bsp: Drehungen

$R = R_\varphi: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ sei die Drehung um den Ursprung um den Winkel φ gegen UZS.



Problem: Berechne $R(u)$ aus $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

Lösung: Polarkoordinaten

A 2D Cartesian coordinate system showing a vector u in the first quadrant. The length is r and the angle with the positive x-axis is α . The vector is labeled $u = \begin{bmatrix} r \cos \alpha \\ r \sin \alpha \end{bmatrix}$.

A 2D Cartesian coordinate system showing a vector $R(u)$ in the first quadrant. The length is r and the angle with the positive x-axis is $\alpha + \varphi$. The vector is labeled $R(u) = \begin{bmatrix} r \cos(\alpha + \varphi) \\ r \sin(\alpha + \varphi) \end{bmatrix}$.

$$\text{Additionstheorem} = \begin{bmatrix} r \cos \alpha \cos \varphi - r \sin \alpha \sin \varphi \\ r \sin \alpha \cos \varphi + r \cos \alpha \sin \varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos \varphi u_1 - \sin \varphi u_2 \\ \sin \varphi u_1 + \cos \varphi u_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}}_{=: A} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\Rightarrow R(u) = Au.$$