5.23 Bsp

$\omega_0, \gamma > 0$

Hatten $\ddot{x}(t) = -\omega_0^2 x(t) - 2\gamma \dot{x}(t)$

$$\Longleftrightarrow \quad \begin{pmatrix} \dot{x}(t) \\ \dot{v}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\omega_0^2 & -2\gamma \end{pmatrix}}_{=: A} \begin{pmatrix} x(t) \\ v(t) \end{pmatrix}$$

$$P_A(\lambda) = \lambda^2 + 2\gamma\lambda + \omega_0^2$$

$$P_A(\lambda) = 0 \Longleftrightarrow \lambda = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

Fall (ii): $\gamma = \omega_0 \Rightarrow \exists_1$ EW $\lambda = -\gamma$.

$$A = \begin{pmatrix} 0 & 1 \\ -\gamma^2 & -2\gamma \end{pmatrix}$$

kein Vielf. von $E \Rightarrow g(-\gamma) < 2$.

(tats. hat $\underline{A - \lambda E} = A + \gamma E = \begin{pmatrix} \gamma & 1 \\ -\gamma^2 & -\gamma \end{pmatrix}$

$\mathrm{Rg} = 1$, also 1-dim Kern $= \mathbb{C}\begin{pmatrix} 1 \\ -\gamma \end{pmatrix}$.)

Also A nicht diagbar.

$$\mathrm{JNF}(A) = \begin{pmatrix} -\gamma & 1 \\ 0 & -\gamma \end{pmatrix}$$

$e^{tA} = ?$

Brauchen: Jordanbasis (w, v)

$w = \mathrm{EV}$, etwa $\begin{pmatrix} 1 \\ -\gamma \end{pmatrix} = w$

Finde v, JNF $\Leftrightarrow$ $Av = w - \gamma v$

$$\Leftrightarrow (A + \gamma E) v = w$$

$$(A - \lambda E) v = w$$

$$\Leftrightarrow \begin{pmatrix} \gamma & 1 \\ -\gamma^2 & -\gamma \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -\gamma \end{pmatrix}$$

$$\Leftrightarrow \gamma v_1 + v_2 = 1$$

$$v_2 = 1 - \gamma v_1$$

Lsg: $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \alpha w$, $\alpha \in \mathbb{C}$ bel.

wähle $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Also: $A = S \, \mathrm{JNF}(A) \, S^{-1}$

mit $S = \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix}$, $\mathrm{JNF}(A) = \begin{pmatrix} -\gamma & 1 \\ 0 & -\gamma \end{pmatrix}$

$$S^{-1} = \frac{1}{\det S} \begin{pmatrix} 1 & -0 \\ -\gamma & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\gamma & 1 \end{pmatrix}$$

$\det S = 1$

$$e^{t\,\mathrm{JNF}(A)} = e^{-t\gamma} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow e^{tA} = e^{-t\gamma} S \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} S^{-1}$$

$$= e^{-t\gamma} \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\gamma & 1 \end{pmatrix}$$

$$= e^{-t\gamma} \begin{pmatrix} 1 - \gamma t & t \\ -\gamma^2 t & 1 + \gamma t \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} = e^{tA} \begin{pmatrix} x(0) \\ v(0) \end{pmatrix} =$$