Anwendung

Satz von Cayley-Hamilton:

$$P_A(A) = 0.$$

Hatten Bew. f. diagbares A

Bew f. $\not$ bel. $A \in M(n, \mathbb{C})$.

$\forall$ Poly P: $\quad P(S \; JNF(A) \; S^{-1})$

$$= S \; P(JNF(A)) \; S^{-1}$$

Block für Block: $\quad P_A(J) = ?$

$$P_A(\lambda) = \det(A - \lambda E) = \det(S^{-1}(A - \lambda E) S) = \det(\underbrace{S^{-1}AS}_{JNF(A)} - \lambda E) =$$

$$= P_{JNF(A)}(\lambda)$$

$$= \prod_{j=1}^{d} (\lambda_j - \lambda)^{a(\lambda_j)} = (-1)^n \prod_{j=1}^{d} (\lambda - \lambda_j)^{a(\lambda_j)}$$

$d = \#$ EWe von A

JK $\quad J = J_m(\lambda_i), \quad m \leq a(\lambda_i)$

$$\Rightarrow P_A(J) = (-1)^n \prod_{j \neq i} (J - \lambda_j E)^{a(\lambda_j)} \; \underbrace{(J - \lambda_i E)^{a(\lambda_i)}}_{0}$$

$J - \lambda_i E = N_m$

$N_m^m = 0 \Rightarrow N_m^{a(\lambda_i)} = 0$

$\Rightarrow P_A(J) = 0 \Rightarrow P_A(A) = 0.$

$\square$