

Groups and Representations

Homework Assignment 12 (due on 21 July 2021)

Problem 42 (continuation of Problem 41)

Let $K : \mathfrak{g} \times \mathfrak{g} \rightarrow \mathbb{R}$ be the Killing form from Problem 41, and let G be such that K is positive definite. We choose an orthonormal basis $\{X_j\}$ with respect to K , i.e. $K(X_j, X_k) = \delta_{jk}$, and define $C_2 \in E(\mathfrak{g})$ by

$$C_2 = \sum_j X_j X_j.$$

Show:

- c) C_2 is independent of the choice of basis.
- d) C_2 is a Casimir operator (the so-called *quadratic Casimir operator*), i.e.

$$\text{Ad}_g(C_2) = C_2 \quad \forall g \in G.$$