

Groups and Representations

Homework Assignment 13 (due on 28 July 2021)

Problem 43

Consider Young diagrams with row lengths $\lambda = (\lambda_1, \dots, \lambda_N)$, and $\lambda' = (\lambda_1+k, \dots, \lambda_N+k)$, $k \geq 1$. Show that the $SU(N)$ -irreps Γ^λ and $\Gamma^{\lambda'}$ are equivalent.

HINT: Use the Littlewood-Richardson rule from Section 7.4 and a result from Section 7.3.

Problem 44

Let Γ^λ be an $SU(3)$ -irrep with Young diagram λ . Determine how often Γ^λ appears in the product $\text{rep } \lambda \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array}$.

HINT: Study separately the cases of rectangular Young diagrams λ (with one or two rows) and of non-rectangular diagrams.

Problem 45

Decompose the product $\text{rep } \square \otimes \square \otimes \square$ of $SU(3)$ into irreps. Use the notation of Problem 28 (e.g. $|uds\rangle = |u\rangle \otimes |d\rangle \otimes |s\rangle \in \square^{\otimes 3}$) and explicitly construct bases for the irreducible invariant subspaces. Compare with the results of Problem 28. What is the relation between the irreducible subspaces with respect to $SU(3)$ and those with respect to S_3 ?