$L: V \to W$

$L': W' \to V'$ duale Abb.

$V, \langle \cdot, \cdot \rangle_V$, $W, \langle \cdot, \cdot \rangle_W$

$L^*: W \to V$ adj. Abb.

<u>A13</u> a) $A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}$ ist inv.bar,

$(A^T A)^{-1} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/6 \end{bmatrix}$

$P = A (A^T A)^{-1} A^T$

$= \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 0 & 1/6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/6 & -1/3 & 1/6 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$Py = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

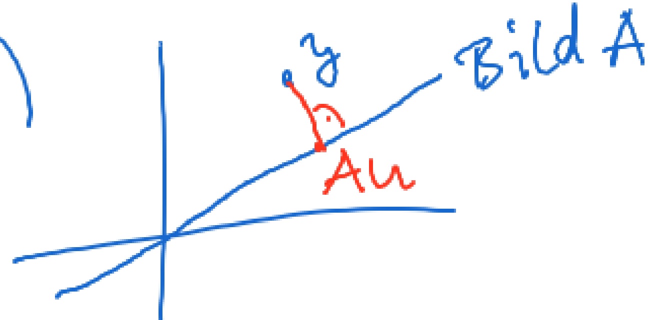
$\exists$ Lsg von $Ax = y$

$\Leftrightarrow y \in \text{Bild } A$

$\Leftrightarrow Py = y$

Hier $Py \neq y$, also nicht lösbar.

b)



$$Au = Py = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$u_1 + u_2 = 2$

$u_1 - 2u_2 = 1$

Lsg $u = \begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}$

$$\|Au - y\| = \|Py - y\| = \left\| \begin{bmatrix} 2-1 \\ 1-1 \\ 2-3 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\| = \sqrt{2}.$$

A15

$$\begin{cases} \dot{x} = Ax \\ x(0) = a \end{cases}$$

hat die Lsg. $x(t) = e^{At} a$

A diagbar, $A = S D S^{-1}$, dann

$$e^{At} = S e^{Dt} S^{-1}.$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$ EWe $\lambda_1 = 1$, $\lambda_2 = 2$

$a(1) = 2$, $a(2) = 1$

$$ER_1 = \mathrm{Kern}(A - E)$$

$$= \mathrm{Kern} \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Rang} = 1}$$

$$= \mathrm{Span}\left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}, \underset{v_3}{\begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}} \right\}$$

$$ER_2 = \mathrm{Kern} \begin{pmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \mathrm{Span} \underset{v_2}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}.$$

$$D = \begin{pmatrix} 1 & & \\ & 2 & \\ & & 1 \end{pmatrix}, \quad e^{Dt} = \begin{pmatrix} e^{t} & & \\ & e^{2t} & \\ & & e^{t} \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

S^{-1} durch Gauß-Jordan

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad R_1 - R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad R_1 - R_3,\ R_2 + R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & & & 1 & -1 & -1 \\ & 1 & & 0 & 1 & 1 \\ & & 1 & 0 & 0 & 1 \end{array}\right) \quad S^{-1}$$

$$x(t) = S e^{Dt} S^{-1} a$$

A11 $\quad A = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

nicht selbstadj.

$A^2 = A$? , $A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \neq A$

$\Rightarrow$ keine Proj.

$A^*A = E$? $A^*A = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E. \Rightarrow A$ unitär, inv.bar

$$\frac{1}{z} = \frac{z^*}{|z|^2} \overset{|z|=1}{=} z^*$$

$V, \langle \cdot , \cdot \rangle$

$L : V \to V$

L s.a. $\iff \forall u, v \in V: \ \langle u, Lv \rangle = \langle Lu, v \rangle$.

$A \in M(n, \mathbb{K})$ s.a. $\iff \overline{A}^T = A$

$\mathcal{B}$ ONB in V

$M_{\mathcal{B}}(L) \cancel{= M_{\mathcal{B}}(L)^*}$ s.a. $\iff L$ s.a.

A1d

B ähnl. $A \iff \exists S \in GL(n, \mathbb{K}):$

$A = \begin{pmatrix} 3 & 3 \\ 0 & 3 \end{pmatrix}$

$B = S A S^{-1}$.

EW $\lambda = 3$.

$\mathrm{Kern}(A - 3E) = \mathrm{Kern} \underbrace{\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}}_{\text{Rang}=1}$ hat $\dim = 1$. $\Rightarrow$ nicht diag.bar

A8

Dimensionsformel

$$\dim(U\cap W) + \overbrace{\underbrace{\dim(U+W)}_{\text{UR von } V}}^{\leq n} = \underbrace{\dim U}_{n-1} + \dim W$$

$$\begin{aligned}\dim(U\cap W) &= \dim U + \dim W - \dim(U+W)\\ &\geq \underbrace{\dim U}_{n-1} + \dim W - n\\ &= \dim W - 1.\end{aligned}$$

A16

$Q = E - 2uu^T$

$Q^T = E^T - 2\underbrace{(uu^T)^T}_{u^{T^T} u^T = u u^T}$

$= E - 2uu^T = Q.$

$Q^T Q = Q^2 = (E - 2uu^T)(E - 2uu^T)$

$= E - 2uu^T - 2uu^T + 4u\underbrace{(u^T u)}_{\|u\|^2 = 1}u^T$

$= E - 2uu^T - 2uu^T + 4uu^T = E.$

A9 $\mathcal{A} = (1, x)$

$\mathcal{B} = (1, x, x^2)$

$p(x) = 1, \; Lp(x) = 4x + 5$

$p(x) = x, \; Lp(x) =$

$3x^2 + 4x^2 + 5x$

$= 7x^2 + 5x$

$$M^{\mathcal{B}}_{\mathcal{A}}(L) = \begin{bmatrix} 5 & 0 \\ 4 & 5 \\ 0 & 7 \end{bmatrix}.$$