Sheet 3 — 13. May 2022

Quantum nonlocality and nonlocal games

Exercise 1: Hidden variable models

Consider two particles in the following state:

$$\rho = \frac{5}{8} |\Psi^-\rangle \langle\Psi^-| + \frac{1}{8} |\Psi^+\rangle \langle\Psi^+| + \frac{1}{8} |\Phi^-\rangle \langle\Phi^-| + \frac{1}{8} |\Phi^+\rangle \langle\Phi^+|$$

The aim of this exercise is to construct a hiddel variable model in the following form: First, we consider a local hiddel variable as the vector

$$\vec{\lambda} = \big(\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)\big) ,$$

for certain angles ϕ and θ. This vector is distributed uniformly on the unit sphere. Now we construct the measurements for both particles.

- For particle A, given any vector $\vec{a}$, the result of the measurement is 1 with probabily $P_A(\vec{a}, \vec{\lambda}) = \cos^2(\alpha_1/2)$, where α_1 is the angle between $\vec{a}$ and $\vec{\lambda}$, and 0 elsewhere.
- For particle B, given any vector $\vec{b}$, the result of the measurement is

$$P_A(\vec{a}, \vec{\lambda}) = \begin{cases} 1 & \text{if } \cos^2(\alpha_2/2) < 1/2 \\ 0 & \text{if } \cos^2(\alpha_2/2) \geq 1/2 \end{cases}$$

  where α_2 is the angle between $\vec{b}$ and $\vec{\lambda}$.

We can ask the following questions:

- Is the state ρ separable?
- Calculate the probability for the outcomes of both measurements for the state ρ.
- Are these probabilities compatible with the hidden variable model?

Exercise 2: CHSH inequality

Consider the CHSH inequality with the following choices for Alice's and Bob's measurements:

- $A_1 = Z_A$.
- $A_2 = \cos(\frac{\pi}{4}) Z_A + \sin(\frac{\pi}{4}) X_A$.
- $B_1 = Z_B$.
- $B_2 = \cos(\frac{\pi}{4}) Z_B - \sin(\frac{\pi}{4}) X_B$.

and the following choice for the state shared by Alice and Bob:

$$\rho = p\frac{I}{4} + (1-p)\left|\phi\right\rangle\left\langle\phi\right| \ ,$$

where $|\phi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.

- First, calculate the CHSH quantity:

  $$\langle\phi|A_1B_1 + A_2B_1 + A_1B_2 - A_2B_2|\phi\rangle$$

  as a function of p. For which values of p is the CHSH inequality violated?

Exercise 3: CHSH game

What are the classical and entangled values of the CHSH game, as introduced in the lectures?

Exercise 4: FFL game

What are the classical and entangled values of the FFL game, as introduced in the lectures?