exercise: Fri 8-10

or 10-12.

homework: hand in electronically via

urm.math.uni-tuebingen.de

Three basic facts of statistical mechanics

1) Heat is unordered motion of the atoms,
   temperatur = intensity
2) A closed system with macro-state ν at t_0 appears macroscopically in the future of t_0 (but not in the past of t_0) like a system with typical micro-state ~~con~~ at t_0 compatible with ν.

3) The real world appears macroscopically like a system that at the big bang is in a typical microstate compatible with V_0, the universe's actual macro-state at the big bang.

Review of Classical Mechanics

<u>Def</u> Classical mechanics: • space = 3d Euclidean space

• time = 1d Euclidean space

• N particles = material points $q_k(t) \in \mathbb{R}^3$

$k \in \{1 \dots N\}$

• eq. of motion

$$m_k \frac{d^2 q_k(t)}{dt^2} = \sum_{j \neq k} G m_j m_k \frac{q_j - q_k}{|q_j - q_k|^3} - \sum_{j \neq k} \frac{e_j e_k}{4\pi\varepsilon_0} \frac{q_j - q_k}{|q_j - q_k|^3}$$

$m_k > 0$, $e_k \in \mathbb{R}$

d-dim

<u>Def</u> Euclidean space = metric space isometric to $\mathbb{R}^d$ with $d(\underline{x},\underline{y}) = |\underline{x}-\underline{y}|$ with

$$|\underline{x}| = \left(\sum_{a=1}^{d} x_a^2\right)^{1/2}$$

metric space (X, d), $d : X \times X \to [0,\infty)$

s.t.
$$d(\underline{x},\underline{y}) = 0 \Leftrightarrow \underline{x} = \underline{y}$$
$$d(\underline{x},\underline{y}) = d(\underline{y},\underline{x})$$
$$d(\underline{x},\underline{z}) \le d(\underline{x},\underline{y}) + d(\underline{y},\underline{z}).$$

isometric $\iff$ $\exists$ isometry

$\varphi : X \to Y$ isometry $\iff$

1) surjective

2) $d_Y(\varphi(\underline{x}), \varphi(\underline{x}')) = d_X(\underline{x}, \underline{x}')$

<u>Conseq</u> φ^{-1} is an isometry

<u>Def</u> Cartesian coordinate system = isometry

$$(X, d) \to (\mathbb{R}^d, d_{Eucl})$$

Prop Eq. of motion of cl. mech. is indep. of the choice of Cartesian coo.

Reason: isometry $\varphi: \mathbb{R}^d \longrightarrow \mathbb{R}^d$

$$\varphi(\underline{x}) = R\,\underline{x} + \underline{a}, \quad \underline{a} \in \mathbb{R}^d, \; R \in O(d)$$

microscopic laws, ODE, 2nd order, non-linear

unknown fct: $t \mapsto q(t) = (q_1(t), \dots, q_N(t))$

trajectory

configuration

initial data: $q(0)$, $\underline{v}(0) = (\underline{v}_1(0), \dots, \underline{v}_N(0))$

$\underline{v}_k(t) = \dfrac{d\, q_k(t)}{dt}$, $\underline{p}_k = m_k \underline{v}_k$ momentum

Q: Is it true that the eq. of motion possesses a unique solution $q(t)$ $\forall t \in \mathbb{R}$ for any given $q(0)$, $v(0)$?

Picard-Lindelöf thm Consider ODE initial value problem

$$\frac{dx}{dt} = F(x, t)$$

$$x(t_0) = x_0$$

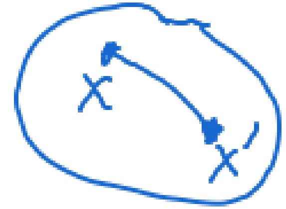
with $\mathbb{R}^n$-valued unknown fct $x(t)$, known $\mathbb{R}^n$-valued F. If F cont and satisfies the Lipschitz cond.

$$\left| F(x', t) - F(x, t) \right| \leq K \, |x' - x|$$

with some $K > 0$ on some open set $\Omega \subseteq \mathbb{R}^{n+1}$, and if $(x_0, t_0) \in \Omega$, then $\exists\, [t_0 - \varepsilon, t_0 + \varepsilon]$ with $\varepsilon > 0$ s.t. $\exists_1$ sol $x(t)$.

Lip $\Rightarrow$ cont.

Lip $\Leftarrow F \in C^1$, ∇F bdd, Ω convex



$$|\nabla F| \leq K$$

reduction of order: introduce $\underline{v}_k = \frac{dq_k}{dt}$

eq. of motion $\Longleftrightarrow \begin{cases} \dfrac{dq_k}{dt} = \underline{v}_k \\[2ex] \dfrac{d\underline{v}_k}{dt} = \dfrac{1}{m_k} \text{force} \end{cases}$

$x = (q, v) \in \mathbb{R}^{6N} = \Gamma$

phase point phase space