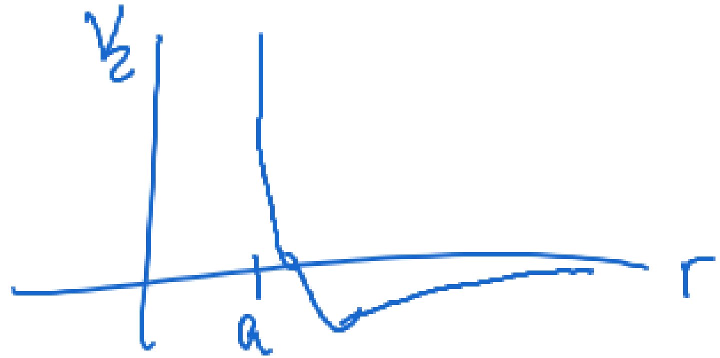
Thms about the thermodyn. limit

$$E, V, N \to \infty, \quad \frac{E}{N} \to e, \quad \frac{V}{N} \to v$$

Assumption 1. technical cond on V_2



Assumption 2. $\Lambda_N \subset \mathbb{R}^3$,

"$\Lambda_N \to \infty$ in the sense of Fisher"

not too much surface compared to the bulk

good

bad

<u>Thm</u> (Ruelle, Fisher) Assumptions 1 & 2,

$\frac{V}{N} \to v$, $\frac{E}{N} \to e$, $H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i<j} V_2(q_i - q_j)$

$\Gamma = (\Lambda_N \times \mathbb{R}^3)^N$,

$$\Gamma_{mc} = \{ x \in \Gamma : E - \Delta E \le H(x) \le E \}, \quad \Delta E > 0 \text{ arbitrary}$$

Then

$$\frac{k}{N} \log \operatorname{vol} \Gamma_{mc} \longrightarrow s(e,v)$$

s is concave, $s : (0,\infty)^2 \longrightarrow [-\infty, \infty)$.

Rem cases in which $\nexists$ thermodyn. lim

a) # pos. charges $\neq$ # neg. charges

b) gravitational interaction

The Boltzmann Equation (1872)

hard sphere gas, $f(q, \underline{v})$

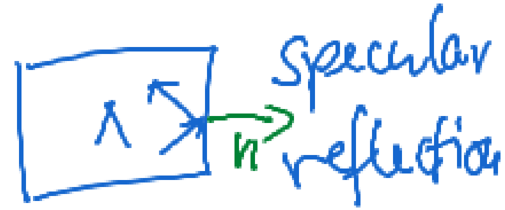
$$\underbrace{\left(\frac{\partial}{\partial t} + \underline{v} \cdot \nabla_{\underline{q}}\right)}_{\text{streaming term}} f(q, \underline{v}, t) = Q(q, \underline{v}, t)$$

collision term $Q(q, \underline{v}, t) = \lambda \int_{\mathbb{R}^3} d^3 \underline{v}_* \int_{S^2} d^2 \underline{\omega} \, \mathbb{1}_{\underline{\omega} \cdot (\underline{v} - \underline{v}_*) > 0} \times$

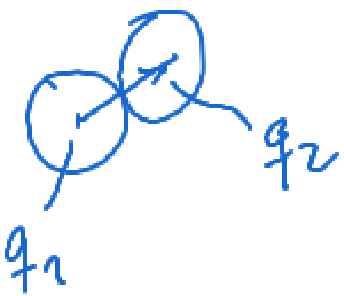
$$\times \, \underline{\omega} \cdot (\underline{v} - \underline{v}_*) \left[f(q, \underline{v}', t) f(q, \underline{v}_*', t) - f(q, \underline{v}, t) f(q, \underline{v}_*, t) \right].$$

$$\underline{v}' = \underline{v} - \left[(\underline{v}-\underline{v}_*)\cdot\underline{\omega}\right]\underline{\omega}$$

$$\underline{v}_*' = \underline{v}_* + \left[(\underline{v}-\underline{v}_*)\cdot\underline{\omega}\right]\underline{\omega} .$$



$$\underline{\omega} = \pm \frac{q_1 - q_2}{2a}$$



$$\Gamma_1 = \Lambda \times \mathbb{R}^3$$

a = radius

boundary condition for $(q,\underline{v}) \in \Gamma_1$ with $q \in \partial\Lambda$

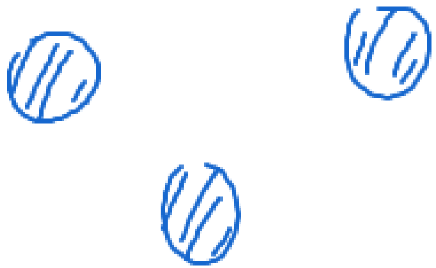
$$f(q,\underline{v},t) = f(q, \underline{v} - 2[\underline{v}\cdot\underline{n}]\underline{n}, t)$$

Boltzmann - Grad limit

$$N \to \infty, \quad a \to 0, \quad 4Na^2 \longrightarrow \lambda$$

volume occupied by the spheres $N \frac{4\pi}{3} a^3 \longrightarrow 0$.

"dilute gas" or "rarefied gas" or "low density limit"



<u>Derivation</u> volume elements

$$\Delta^3 q \, \Delta^3 \underline{v} \longrightarrow d^3 q \, d^3 \underline{v}$$

Claim: $N \, Q(q, \underline{v}) \, d^3 q \, d^3 \underline{v}$ = rate of change in particle number in $d^3 q \, d^3 \underline{v}$

= gain − loss

no. of collisions within dt of two balls in d^3q with velocities in $d^3\underline{v}$ and $d^3\underline{v}_*$ and collision parameter in $d^2\underline{\omega}$ $= \mathcal{N}$

<u>Claim</u> $\mathcal{N} = 4N^2a^2\,\mathbb{1}_{\underline{\omega}\cdot(\underline{v}-\underline{v}_*)>0}\,\underline{\omega}\cdot(\underline{v}-\underline{v}_*)\,f(q,\underline{v})\times f(q,\underline{v}_*)\,d^3q\,d^3\underline{v}\,d^3\underline{v}_*\,\underbrace{d^2\underline{\omega}}_{dt}$

Stoßzahlansatz
hypothesis of molecular chaos.
$\Longleftarrow$ micro positions are as if random and independent.

gains: $(\underline{v}, \underline{v}_*) \xmapsto{R_{\underline{\omega}}} (\underline{v}', \underline{v}_*')$ given $\underline{\omega}$

$\mathbb{R}^6 \to \mathbb{R}^6$

Preparation: Proposition 3.

a) $R_{\underline{\omega}} \in O(6)$

b) $\det R_{\underline{\omega}} = -1$

c) $R_{\underline{\omega}}^2 = \underline{I}$

d) $R_{-\underline{\omega}} = R_{\underline{\omega}}$

e) $\underline{\omega} \cdot (\underline{v}' - \underline{v}_*') = -\underline{\omega} \cdot (\underline{v} - \underline{v}_*)$

(HW)

$$\text{gains} = 4N^2a^2\, d^3q\, dt \int_{S^2} d^2\underline{\omega} \int_{\mathbb{R}^6} d^6u' \, 1\, 1_{\underline{\omega}\cdot(\underline{v}-\underline{v}_*)>0}\, \underline{\omega}\cdot(\underline{v}-\underline{v}_*) \times$$

with $u'\in C$

$$f(q,\underline{v},t)\, f(q,\underline{v}_*,t)\, 1_{\cancel{R_{\underline{\omega}}} u \in C}$$

u'

change of variables $u' = R_\omega u$, $\qquad R_\omega^{-1} = R_\omega$

$$= +4N^2q^2 dq^3\, dt \int d^2\underline{\omega} \int_C d^6u \;\; 1_{\underbrace{\underline{\omega}\cdot(\underline{v}'-\underline{v}_*')}_{-\underline{\omega}\cdot(\underline{v}-\underline{v}_*)} \cancel{>0}}\; \underbrace{\underline{\omega}\cdot(\underline{v}'-\underline{v}_*')}_{}$$

$-\underline{\omega}$, $+\underline{\omega}$

$$f(q,\underline{v}')\, f(q,\underline{v}_*')\, \cancel{1_{u\in C}}$$

$$= \text{term in } Q. \qquad \square$$

$$\text{so} \quad Q = \text{gains} - \text{losses} \,/\, N\, d^3q\, d^3\underline{v}$$