Lanford's Thm

Validity of the Boltzm. Eq.

Boltzm.-Grad limit

$$N \to \infty, \quad a \to 0, \quad 4Na^2 \longrightarrow \lambda.$$

Very brief version:

"most $X \in \Gamma_{f_0}$ evolve so that $X_t \in \Gamma_{f_t}$,

where $t' \mapsto f_{t'}$ is the sol. of the Boltzm. Eq.*,

at least for $t \in [0, \frac{1}{5}\bar{t})$."

Brief version:

For every large no. N of hard spheres and $4Na^2 = \lambda$, for every $t \in [0, \frac{1}{5}\bar{t})$, for any nice density f_0 in $\Gamma_1 = \Lambda \times \mathbb{R}^3$ and a partition of Γ_1 into cells C_i that are small but not too small, most x with emp. distr. f_0 (rel. to C_i within small tolerances) evolve in such a way that the emp. distr. of x_t is close to f_t.

Lanford's thm (1974-76) $\Gamma_1 = \Lambda \times \mathbb{R}^3$, $\Lambda \subseteq \mathbb{R}^3$ compact with smooth $\partial\Lambda$. $f_0 : \Gamma_1 \to [0, \infty)$

- cont., satisfies b.c., normalized,
- has compact supp. $\Lambda \times \Sigma$ with compact $\Sigma \subseteq \mathbb{R}^3$.
- $\cancel{\exists}\, z, \beta > 0$ s.t. $f_0(q, \underline{v}) \leq z\, e^{-\beta m \underline{v}^2/2}$

$$\bar{t} = \left(\frac{m\beta}{3}\right)^{1/2} \frac{1}{\pi \lambda z}$$

Let $\Delta f^{(N)} > 0$ so that $\Delta f^{(N)} \longrightarrow 0$ as $N \to \infty$ (but not too fast)

$N \Delta f^{(N)} \longrightarrow \infty$. $\mathcal{A}^{(N)} = \{ A_i^{(N)} : i = 1 \ldots \nu_N \}$ partition of $\Lambda \times \Sigma$ so that the cells $A_i^{(N)}$ shrink to 0 but not too fast:

- $\max_{i \le r_N} q\text{-diam}(A_i^{(N)}) \xrightarrow{N\to\infty} 0$

- same v

- $N \Delta f^{(N)} \quad \inf_{i \le r_N} \text{vol}(A_i^{(N)}) \longrightarrow \infty$

- $\max_{i \le r_N} \text{vol}(\partial_a A_i) / \text{vol}(A_i) \longrightarrow 0$

$N_i(x) = \#\{j : x_j \in A_i\}$ occ. no. Set

$$\Gamma^{(N)}_{f_0} := \left\{ x \in \Gamma_1^N : \left| \frac{N_i(x)}{N} - \int_{A_i} dx_1\, f_0(x_1) \right| \leq \mathrm{vol}(A_i)\, \Delta f^{(N)} \right\}.$$

$\mu^{(N)}$ = uniform over $\Gamma^{(N)}_{f_0}$, i.e.,

$$\mu^{(N)}(B) = \frac{\mathrm{vol}_\Gamma\left(B \cap \Gamma^{(N)}_{f_0}\right)}{\mathrm{vol}_\Gamma\left(\Gamma^{(N)}_{f_0}\right)}$$

Then $\forall t \in [0, \frac{1}{5}\bar{t})$, $\forall \varepsilon > 0$, $\forall A \subseteq T_1$

$$\mu^{(N)} \left\{ X : \left| \frac{N_A(T^t X)}{N} - \int_A dx_1 \, f_t(x_1) \right| < \varepsilon \right\} \longrightarrow 1$$

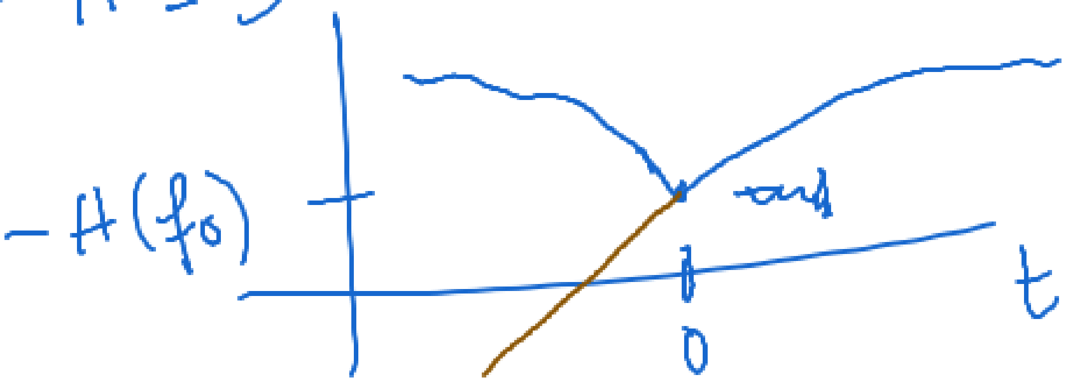
as $N \to \infty$.

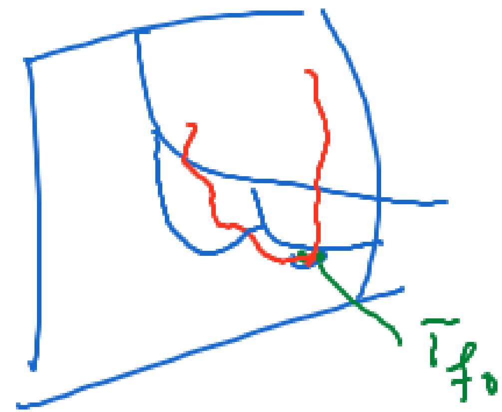
negative t: $\quad x^r = (q, -v) \qquad x = (q, v)$

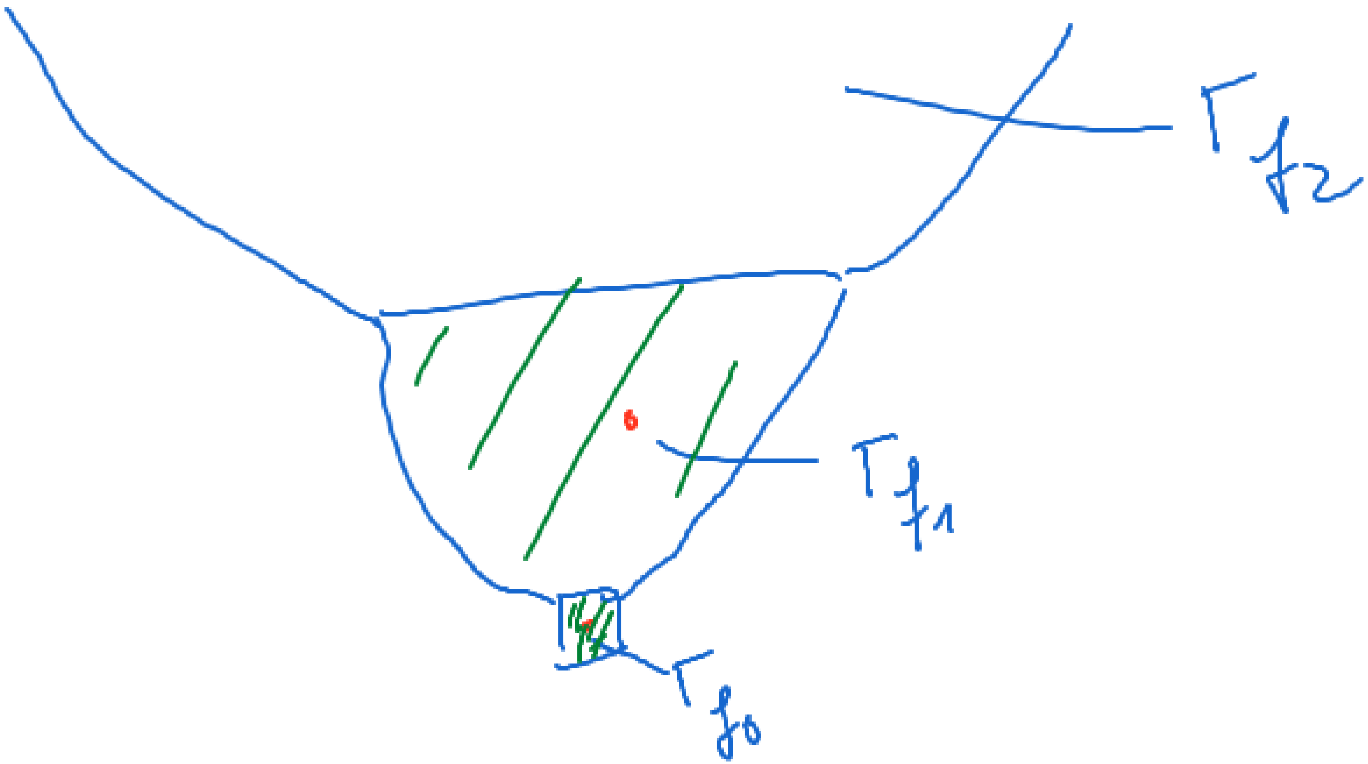
$$f^r(q, \underline{v}) = f(q, -\underline{v})$$

$$(((f_0)^r)_t)^r = \text{sol. of Boltzm. Eq. with } \lambda \to -\lambda$$

$-H = S$







Γ_{f_2}
Γ_{f_1}
Γ_{f_0}

Chap. 9 : The Thermodyn. Arrow of Time