MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
DER UNIVERSITÄT TÜBINGEN

WiSe 2022/23

Exercises: Mathematical Statistical Physics

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Sheet 2

Exercise 1: (Kolmogoroff's Axioms) Let Ω be a finite set, $P : \mathfrak{P}(\Omega) \mapsto \mathbb{R}$ a probability measure and $A, B \in \mathfrak{P}(\Omega)$. Prove that $\forall A, B \in \mathfrak{P}(\Omega)$

(a) $P(A) \leq 1$

(b) $P(A^c) = 1 - P(A)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Clarify with each equation which of Kolmogoroff's axiom is used!

Exercise 2: (Independence) On the Laplace space $\Omega = \{\omega = (i, j) \mid i, j \in \{1, 2, 3\}\}$ (e.g. throwing a dice with only three sides) consider the following two random variables:

$$X : \Omega \to \{0, 1\} \text{ with } \begin{cases} X(i, j) = 0 & \text{if } |i - j| \geq 1 \\ X(i, j) = 1 & \text{else} \end{cases}$$

and

$$Y : \Omega \to \{0, 1\} \text{ with } \begin{cases} Y(i, j) = 0 & \text{if } j = 3 \\ Y(i, j) = 1 & \text{else} \end{cases}$$

Show that the random variables X and Y are stochastically independent. The random variables X, Y are called stochastically independent if for all possible combinations $x, y \in \{0, 1\}$ it holds that:

$$P(\{\omega | X(\omega) = x\} \cap \{\omega | Y(\omega) = y\}) = P(\{\omega | X(\omega) = x\}) \cdot P(\{\omega | Y(\omega) = y\})$$

Exercise 3: Prove that the efficiency of the carnot process for given temperature T_C for the cold reservoir and T_H for the hot reservoir is given by $1 - \frac{T_C}{T_H}$.

Exercise 4: Prove the validity of Louiville's Theorem in the presence of an interaction with an external magnetic field, i.e. a force of the form $F = \sum_{j=1} B(x_j) \times p_j$ for an arbitrary $B : \mathbb{R}^3 \to \mathbb{R}^3$.