Exercises: Mathematical Statistical Physics

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Sheet 7

Exercise 1: (Transition matrix)
Calculate the transition matrix of the following Markov chains:

(a) A total of $2n$ atoms with n atoms denoted Ä"(=atoms) and "R"(=rest), respectively, are placed separetly in two boxes, and at each stage an atom from each box is selected at random and they are interchanged. The state of this system is the atom number Äin the box which were initially filled with Ä".

(b) A total of n atoms are placed in two boxes, denoted "L"(=left) and "R"(=right), respectively, and at each stage an atom is selected at random and a box is selected at random to put the atom inside, where the selection of the box is Bernoulli distribued. The state of this system is the atom number in the box "L".

Exercise 2: (Recurrence)
Show that any state $k \in E$ of a finite closed communication class is recurrent. Further show that no state $k \in E$ of a finite and open communication class is recurrent.

Exercise 3: (Transience and Recurrence)
Prove the following statements:

a) If $p = \frac{1}{2}$, then any state of the simple random walk is recurrent.

b) If $p \neq \frac{1}{2}$ then any state is transient.

c) The simple symmetric random walk on $\mathbb{Z}^d$ is recurrent if and only if $d = 1, 2$ and transient if and only if $d \geq 3$.