MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
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Exercises: Mathematical Statistical Physics

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Sheet 8

Exercise 1: (Ehrenfest model)
Consider the Ehrenfest model. Assume that $\alpha_k^0 = 1$ for some $k \in \{0, 1, \dots, N\}$. Show that for all $j \in \{0, 1, \dots, N\}$ the limit $\lim_{t\to\infty} \alpha_j^t$ does not exist.

Exercise 2: (Irredcible Markov Chain)
Consider an irreducible Markov chain. Then the following statements are equivalent:

(1) Some state is positive recurrent.

(2) All states are positive recurrent.

(3) The chain has invariant probability measure $\lambda = (\lambda_i)_{i\in E}$.

Prove if these statements hold, then $\mathbb{E}(T_i) = \frac{1}{\lambda_i}$ for all $i \in E$.

Exercise 3: (Transience)
Let $j, k \in E$, let $N_k := \sum_{j=1}^{\infty} \mathbb{1}_{X_n=k}$. Show that k is transient iff

$$\mathbb{P}\left(N_k < \infty \mid X_0 = j\right) = 1$$