Exercises: Mathematical Statistical Physics

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Sheet 9

Exercise 1: (Stationary Measure, irreducibility, periodicity)
Let X be a Markov chain with transition matrix

$$\Gamma = \begin{pmatrix} 1-p & \frac{p}{2} & \frac{p}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

for $p \in [0,1]$.

a) Find every stationary measure of the Markov chain.

b) Find all p such that X is irreducible.

c) Find all p such that X is aperiodic.

Exercise 2: (Irreducible, aperiodic Markov chain)
Let X be a aperiodic Markov chain with state space $E = \{s_1, s_2, \dots s_k\}$. Prove that for all $s_i, s_j \in E$

a) $\mathbb{P}(T_{ij} < \infty | X_0 = s_i) = 1$

b) $\mathbb{E}(T_{ij} | X_0 = s_i) < \infty$.

Exercise 3: (Stationary measure)
Let X be a irreducible Markov chain with state space $E = \{s_1, s_2, \dots, s_n\}$ and transition matrix P. Show that if there exists a state s_i with $P_{ii} > 0$ then X is aperiodic.

Exercise 4: Let X be a Markov chain modelling the movement of an chess piece on a chessboard (X_t shell denote the position at time t) with state space E and transition matrix P (chess piece chooses one of all possible moves with equal probability, chess piece moves every time). Classify whether the Markov chain is irreducible or aperiodic if the chess piece is a

a) King

b) Bishop (Runner)

c) Knights.