MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
DER UNIVERSITÄT TÜBINGEN SoSe 2026

Mathematical Statistical Physics

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Sheet 1

Exercise 1: Let Ω be a set and $\mathcal{A}$ be a corresponding σ-algebra. Show that:

(a) For any two probability measures $\mathbb{P} : \mathcal{A} \to \mathbb{R}$ and $\mathbb{Q} : \mathcal{A} \to \mathbb{R}$, and $0 \leq a \leq 1$, $\mathbb{S} := a\mathbb{P} + (1-a)\mathbb{Q}$ is also a probability measure on $\mathcal{A}$.

(b) Every probability measure $\mathbb{P} : \mathcal{A} \to \mathbb{R}$ is monotonically increasing in the sense that $A \subset B \Rightarrow \mathbb{P}(A) \leq \mathbb{P}(B)$.

Exercise 2: Let $\mathcal{A} \subset \mathcal{P}(\mathbb{R})$ be defined by $A \in \mathcal{A} \Leftrightarrow A$ or A^c is countable. Show that:

(a) $\mathcal{A}$ is a σ-algebra.

(b) $\mathcal{A}$ is the σ-algebra generated by all singleton subsets of $\mathbb{R}$.

Exercise 3: Let Ω be a set and, for some index set I, $\mathcal{A}_i$ be σ-algebras with respect to Ω for any $i \in I$.

(a) Show that $\bigcap_{i \in I} \mathcal{A}_i$ is a σ-algebras with respect to Ω.

(b) Show that $\bigcup_{i \in I} \mathcal{A}_i$ is not necessarily a σ-algebras with respect to Ω.

Exercise 4: Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space. Show that the properties

a) A is independent of B and A is independent of $C \implies A$ is independent of $B \cap C$

b) A is independent of B and A is independent of $C \implies A$ is independent of $B \cup C$

do not hold in general.
What is the situation under the additional assumption that $A \cap B = \emptyset$?

Please submit the exercise sheet in pairs or groups of three via URM by 2:00 PM on May st, 2026.