MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
DER UNIVERSITÄT TÜBINGEN SoSe 2026

Mathematical Statistical Physics

Prof. Dr. P. Pickl

Sheet 11

In class we showed that the Bose-Einstein and Fermi-Dirac distributions are given by

$$\frac{N_i}{g_i} = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} \pm 1} \tag{1}$$

where the $(+)$ sign corresponds to Fermi-Dirac, the $(-)$ sign corresponds to Bose-Einstein, ϵ_i is the energy of state i, μ is the chemical potential, k_B is the Boltzmann constant, and T is the temperature.

Exercise 1: The Fermi-Dirac distribution describes fermions (e.g., electrons) which obey the Pauli exclusion principle. Let μ_0 be the chemical potential at absolute zero ($T = 0$ K), widely known as the Fermi Energy (E_F).

Use formula (1) to calculate the average occupancy $\frac{N_i}{g_i}$ for a state with energy ϵ_i at $T = 0$ K for two distinct cases:

1. $\epsilon_i < E_F$
2. $\epsilon_i > E_F$

Sketch or describe what the occupancy function $f(\epsilon)$ looks like as a function of energy at $T = 0$ K versus a small non-zero temperature $T > 0$ K.

Exercise 2: In class we calculated the variance in the potential $\sum_{j=1}^{N} V(x_j)$ for an N-body Fermi gas at zero temperature. The goal of this exercise is to better understand this calculation in a simplified setting.

Repeat the calculation of the variance in the potential $\sum_{j=1}^{N} V(x_j)$ in the case $N = 3$, i.e. when Ψ is given by the anti-symmetrized product of the states $\frac{1}{\sqrt{2\pi}}$; $\frac{e^{ikx}}{\sqrt{2\pi}}$ and $\frac{e^{iikx}}{\sqrt{2\pi}}$.

Exercise 3: Show that in the high-temperature limit both distributions given in (1) reduce to the classical Maxwell-Boltzmann distribution:

$$\frac{N_i}{g_i} \approx e^{-(\epsilon_i - \mu)/k_B T} \tag{2}$$

Please submit the exercise sheet in pairs or groups of three via URM by 2:00 PM on July 17^{th}, 2026.