

Mathematical Statistical Physics

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Sheet 1

Exercise 1: A continuously distributed random variable X is said to be exponentially distributed with parameter $\lambda > 0$ if the corresponding density ρ is given by

$$\rho_\lambda(x) = \begin{cases} 0 & \text{if } x < 0 \\ \lambda e^{-\lambda x} & \text{otherwise.} \end{cases}$$

- (a) Show that ρ_λ is indeed a probability density.
- (b) Determine the expectation value of ρ_λ .
- (c) Show that for any $a, \delta > 0$, the conditional probability $\mathbb{P}_{X \geq a}(X \in [a, a + \delta])$ does not depend on the parameter a .

Exercise 2: Show that: The Borel σ -algebra is the σ -algebra generated by all closed intervals.

Exercise 3: Let X and Y be random variables, each capable of taking on only two distinct values. Show that

$$X, Y \text{ are uncorrelated} \Leftrightarrow X, Y \text{ are independent.}$$

Find an example of random variables $X : \Omega \rightarrow \{0, 1\}$ and $Y : \Omega \rightarrow \{-1, 0, 1\}$, that are uncorrelated but not independent.

Please submit the exercise sheet in pairs or groups of three via URM by 2:00 PM on May 8th, 2026.