

# Mathematical Statistical Physics

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## Sheet 3

**Exercise 1:** In class we discussed a heuristic explanation for validating the Moivre-Laplace Theorem. Generalize this to the case  $p \neq 1/2$ .

**Exercise 2:** Let  $g_t(x) := C_t e^{-\frac{x^2}{2\sigma_t^2}}$ . Find  $C_t$  and  $\sigma_t$  such that  $g_t$  solves the heat equation

$$\frac{d}{dt}g_t = \Delta g_t.$$

Show that  $\rho \star g_t$  also solves the heat equation for any  $\rho \in L^1$ .

**Exercise 3:** Consider the random walk we introduced in class in one dimension. Assume that for each time step, the probability to make a step to the left is given by  $p$ , the probability to make a step to the right is given by  $q$  and the probability to stay at the same position is given by  $k$ . Of course  $1 = p + q + k$ .

Use the Central Limit Theorem to find a continuous function  $f_t(x)$  that approximates the probability function for large  $t$ . What differential equation does it solve?

Please submit the exercise sheet in pairs or groups of three via URM by 2:00 PM on May 15<sup>th</sup>, 2026.