MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
DER UNIVERSITÄT TÜBINGEN

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Mathematical Statistical Physics

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Sheet 8

Exercise 1: Given a system of N particles of equal mass on a three-dimensional torus $[0,1)^3$ subject to Newtonian dynamics with Newtonian gravitation. Let $E > 0$ be the total energy of the system.

(a) Let $N = 2$. Show that there is an invariant, normalizable measure on the energy shell with energy E.

Hint: Calculate for any $\Delta x \in [0,1)$ the kinetic energy of the two particles assuming that their distance equals Δx, and evaluate the Lebesgue measure of the phase space volume corresponding to these configurations.

(b) Show that the Lebesgue measure of the energy shell volume is not bounded in the case $N > 3$.

Exercise 2: Consider a container divided into two equal compartments, each of volume V. The system is kept at a constant temperature T.

- **Case A:** The left compartment contains N molecules of Gas 1, and the right compartment contains N molecules of a *different* Gas 2.
- **Case B:** Both compartments contain N molecules of the *same* Gas 1.

When the partition between the two compartments is removed, the gases mix.

Calculate the change in entropy when the gases mix in both cases.

Exercise 3: Consider a system of N particles subject to free Newtonian motion, i.e. for any $t \in \mathbb{R}$ and any $j \in \{1, 2, \dots, N\}$ $x_t^j = x_0^j + v^j t$ and $v^j = \mathit{const}$ for any $t \in \mathbb{R}$ and any $j \in \{1, 2, \dots, N\}$.

Assume that initially the macro-state of the system is defined via the phase-space density $\rho_0(x, v)$.

(a) Show that the entropy of the system stays constant

(b) Show that the local temperature of the gas (i.e. the variance of the velocities) tends to zero as $N \to \infty$.

Please submit the Exercise sheet in pairs or groups of three via URM by 2:00 PM on June 26^{th}, 2026.