MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
DER UNIVERSITÄT TÜBINGEN

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Mathematical Statistical Physics

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Sheet 5

Exercise 1: Let ρ_t be the solutuion of the heat equation given by

$$\rho_t = Ct^{-1/2}e^{\frac{-x^2}{2t}}$$

Calculate the entropy of ρ_t as a function of time.

Exercise 2: Consider two boxes of identical material and size. Assume that the two boxes are isolated and that one of the boxes has temperature T_1 while the other one has temperature T_2. Calculate the change in entropy when the isolation between the boxes is removed and both boxes reach thermal equilibrium.

Exercise 3: Consider a system of N distinguishable, non-interacting particles in a box. The energy levels available to each particle are evenly spaced and unbounded:

$$\epsilon_m = m\epsilon_0 \quad \text{where} \quad m \in \{0, 1, 2, \dots\}$$

The system has a fixed total internal energy E, which is an integer multiple of the fundamental energy unit ϵ_0, such that $E = Q\epsilon_0$ for some integer $Q \geq 0$. Here, Q represents the total number of indistinguishable energy quanta distributed among the N particles.

(a) Using a combinatorial argument, derive the exact number of unique microstates $\Omega(N, Q)$ capable of realizing this macrostate.

(b) Write down the exact expression for the Boltzmann entropy $S(N, Q)$ of the system.

(c) Assume both the number of particles and the number of energy quanta are extremely large ($N \gg 1, Q \gg 1$). Apply Stirling's approximation ($\ln x! \approx x \ln x - x$) to find a simplified expression for the entropy. Express the final entropy per particle (S/N) as a function of the average number of energy quanta per particle, defined as $q = Q/N$.

Please submit the exercise sheet in pairs or groups of three via URM by 2:00 PM on June 5^{th}, 2026.