

Mathematical Quantum Theory
Exercise sheet 1
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Exercise 1. [10 points] Consider a quantum system, whose wave function $\psi(t)$ at the time t is a vector in \mathbb{C}^2 . Suppose that the Hamiltonian is given by:

$$H = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}. \quad (1)$$

Find the solution of the initial value problem:

$$\begin{aligned} i\dot{\psi}(t) &= H\psi(t) \\ \psi(t_0) &= \psi_0. \end{aligned} \quad (2)$$

Exercise 2. [10 points] Let $f \in L^1(\mathbb{R}) \cap C^1(\mathbb{R})$, such that $f' \in L^1(\mathbb{R})$.

(a) Show that $f \in C_\infty(\mathbb{R})$.

(b) Show that, for $g \in L^\infty(\mathbb{R}) \cap C^1(\mathbb{R})$ and $g' \in L^\infty(\mathbb{R})$, the following identity holds true:

$$\int_{\mathbb{R}} dx g(x) f'(x) = - \int_{\mathbb{R}} dx g'(x) f(x). \quad (3)$$

(c) Prove that $\widehat{f'}(k) = ik\widehat{f}(k)$ for all $k \in \mathbb{R}$.

Exercise 3. [10 points] Let $n \in \mathbb{N}$ and $f \in L^1(\mathbb{R})$. Find conditions that imply the respective statements.

(a) $\widehat{f} \in C^n(\mathbb{R})$.

(b) $\sup_{k \in \mathbb{R}} ||k|^n \widehat{f}(k)| < \infty$.

(c) $\widehat{f} \in L^1(\mathbb{R})$.

Exercise 4. [10 points] Let $a, b \in \mathbb{C}$ with $\operatorname{Re} a > 0$ and let $f : \mathbb{R} \rightarrow \mathbb{C}$ defined as $f(x) = e^{-ax^2/2+bx}$. Show that $f \in \mathcal{S}(\mathbb{R})$ and compute $\widehat{f}(k)$.