Mathematical Quantum Theory
Exercise sheet 6
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Exercise 1. [14 points] Let $\mathcal{H}_1$, $\mathcal{H}_2$ be Hilbert spaces, and let $U : \mathcal{H}_1 \to \mathcal{H}_2$ unitary. Suppose that $(H, D(H))$ is a selfadjoint operator on $\mathcal{H}_1$, $D(H) \subset \mathcal{H}_1$, $H : D(H) \to \mathcal{H}_2$. Prove that:

$$(UHU^*, UD(H)) \qquad \text{is selfadjoint on } \mathcal{H}_2. \tag{1}$$

Exercise 2. [14 points] Let $M \subset \mathcal{H}$. Prove that:

$$\overline{M}^\perp = M^\perp \ , \qquad M^{\perp\perp} = \overline{M} \ . \tag{2}$$

Exercise 3. [12 points] Prove that $(-\Delta, H^2(\mathbb{R}^d))$ is selfadjoint on $L^2(\mathbb{R}^d)$.

[Hint. Use Exercise 1, together with the fact that the multiplication operator A_f, $(A_f\psi) = f(x)\psi(x)$, for $f : \mathbb{R}^d \to \mathbb{C}$ is selfadjoint if and only if f is real valued.]

Exercise 4. [10 points, bonus] Let $\Omega \subseteq \mathbb{R}^d$ open, and $f \in C(\Omega)$. Suppose that for every $\varphi \in C_c^\infty(\Omega)$ we have:

$$\int_\Omega dx\, f(x)\varphi(x) = 0 \ . \tag{3}$$

Prove that $f = 0$.