Introduction to Partial Differential Equations
Exercise sheet 5
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Exercise 1. [10 points]

1) Show that the general solution of the equation $u_{xy} = 0$ is of the form:

$$u(x, y) = F(x) + G(y)$$

for arbitrary F, G.

2) Using the change of variables $\xi := x + t$ and $\eta := x - t$,show that $u_{tt} - u_{xx} = 0$ if and only if $u_{\xi\eta} = 0$.

3) Use Part 1 and Part 2 to rederive d'Alembert's formula.

4) Find under what conditions on the initial datum $u = g$ and $u_t = h$ on $\mathbb{R} \times \{t = 0\}$, the solution $u(x, t)$ is a right-moving wave, resp. a left-moving wave.

Exercise 2. [10 points]

Find a solution of the following Cauchy problem:

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{for } x \in \mathbb{R},\, t > 0 \\ u(x, 0) = g(x) & \text{for } x \in \mathbb{R} \\ u_t(x, 0) = h(x) & \text{for } x \in \mathbb{R} \end{cases}$$

with initial datum:

$$g(x) = 0, \qquad h(x) := \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$$

Exercise 3. [10 points]

Let $f \in C(\mathbb{R})$, and consider the following boundary value problem:

$$\begin{cases} u_{tt} - u_{xx} = f(x) \sin t & \text{for } (x, t) \in \mathbb{R} \times (0, \infty) \\ u(x, 0) = u_t(x, 0 = 0 & \text{for all } x \in \mathbb{R} \end{cases}$$

Find a particular solution of the problem, and prove uniqueness.