# Introduction to Partial Differential Equations
## Exercise sheet 7
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**Exercise 1. [10 points]**

Consider the initial value problem

$$\begin{cases} u_{tt} - \Delta u + u_t + u = 0 & \text{in } \mathbb{R}^n \times \{t > 0\} \\ u(x,0) = f(x) \end{cases}$$

By using the Fourier transform, show that for any $f \in L^2(\mathbb{R}^n)$ the equation admits a solution $u(x,t)$ such that $\|u(\cdot,t)\|_2 \leq C$ for all $t \in (0,\infty)$.

**Exercise 2. [10 points]**

Consider the following initial value problem:

$$\begin{cases} \partial_t u - \partial_x^2 u - 2x\partial_x u = 0 & x \in \mathbb{R}, t > 0 \\ u(x,0) = g(x), \end{cases}$$

where $g \in C(\mathbb{R}) \cap L^2(\mathbb{R})$. Solve the equation using Fourier transform.

**Exercise 3. [10 points]**

Consider the initial value problem:

$$\begin{cases} u_t - xu_x + u - 1 = 0 & x \in \mathbb{R}, t > 0 \\ u(x,0) = \sin x & x \in \mathbb{R} \end{cases}$$

Find a solution using the method of characteristics. Discuss the behavior of $\|u_x(\cdot,t)\|_\infty$ as $t \to \infty$.