Introduction to Partial Differential Equations
Exercise sheet 8
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Exercise 1. [10 points]

1) Consider the initial value problem:

$$\begin{cases} u_t + \partial_x \frac{u^2}{2} = 0 & x \in \mathbb{R}, t > 0 \\ u(x,0) = g(x) & x \in \mathbb{R}, \end{cases} \tag{1}$$

with $g(x)$ given by

$$g(x) = \begin{cases} 2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the solution using the method of the characteristics. Find the expression for the shock curve.

2) Consider the PDE in Eq. (1), this time with initial datum

$$g(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$$

Find the solution using the method of the characteristics. Find the expression for the shock curve.

Exercise 2. [10 points]

Consider the initial value problem:

$$\begin{cases} u_t + u^3 u_x = 0 & x \in \mathbb{R}, t > 0 \\ u(x,0) = g(x) & x \in \mathbb{R}, \end{cases}$$

with, for $a > 0$, $g(x)$ given by

$$g(x) = \begin{cases} a(1 - e^x) & \text{if } x < 0 \\ -a(1 - e^x) & \text{if } x > 0 \end{cases}$$

Find the smallest time for which a shock appears.

Exercise 3. [10 points]

Consider the initial value problem:

$$\begin{cases} u_t + (1 - 2u)u_x = 0 & x \in \mathbb{R}, t > 0 \\ u(x,0) = g(x) & x \in \mathbb{R}, \end{cases}$$

with, for $0 < a < 1$,

$$g(x) = \begin{cases} a & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Determine the characteristics and find a solution. Are there shock lines?