Exercises for "Wave Equations of Relativistic Quantum Mechanics"

Preparatory Sheet

Winter Semester 2018/19

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Exercise 1. *Index notation.*

(a) In four dimensions, calculate $\delta^{\mu}{}_{\mu}$ and $\eta^{\mu}{}_{\mu}$.

(b) For 4-vectors $x = (x^0, x^1, x^2, x^3)$ and $a = (a^0, a^1, a^2, a^3)$, calculate the expressions

$$\partial_\mu x^\nu, \quad \partial_\mu x_\nu, \quad \partial^\nu x_\mu, \quad a^\mu \partial_\mu x^\nu, \quad \partial_\mu x^2, \quad \partial_\mu \sqrt{x^2}, \quad \partial_\mu \partial^\mu x^2. \tag{1}$$

(c) Let A, S be two tensors with $A_{\mu\nu} = -A_{\nu\mu}$ and $S_{\mu\nu} = S_{\nu\mu}$. What is the result of the contraction $A_{\mu\nu} S^{\mu\nu}$?

(d) Reformulate the continuity equation

$$\partial_t \rho(t, \mathbf{x}) + \operatorname{div} \mathbf{j}(t, \mathbf{x}) = 0 \tag{2}$$

as an equation for a 4-vector using index notation. (Here, div is to be understood with respect to the spatial variables only.)