Universität Tübingen, Fachbereich Mathematik
Dr. Stefan Keppeler & Anna Sancassani Wintersemester 19/20

Groups and Representations

Homework Assignment 1 (due on 23 Oct 2019)

Problem 1

For the group table of a finite group prove that no two elements within one row (or column) can be the same.

Problem 2

List all possible groups (up to isomorphy) of order 3 and 4 by explicitly constructing their group tables.

Problem 3

Let $(G, \circ)$ and $(G', \bullet)$ be groups and let $f : G \to G'$ be a homomorphism. Show that f maps the identity in G to the identity in G', i.e. $f(e_G) = e_{G'}$.

Problem 4

Show that the map $\exp : (\mathbb{R}, +) \to (\mathbb{C}\backslash\{0\}, \cdot)$, $t \mapsto \mathrm{e}^{2\pi \mathrm{i} t}$ is a group homomorphism. Determine kernel and image of exp, and check whether these are subgroups of $(\mathbb{R}, +)$ and $(\mathbb{C}\backslash\{0\}, \cdot)$, respectively.

Problem 5

Let

$$\begin{aligned}\mathcal{B} &= \{f : \mathbb{R} \to \mathbb{R} | f \text{ bijective}\} \quad \text{and} \\ \mathcal{A} &= \{f : \mathbb{R} \to \mathbb{R} | \exists a, b \in \mathbb{R}, a \neq 0 : f(x) = ax + b\} \ .\end{aligned}$$

Show that $(\mathcal{B}, \circ)$, with $\circ$ the composition of functions, is a group and that $\mathcal{A}$ is a subgroup of $\mathcal{B}$. Is $\mathcal{B}$ or $\mathcal{A}$ abelian?

`www.tinyurl.com/ws1920-grar`