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Foundations of Quantum Mechanics: Assignment 5

Exercise 18: Essay question. Describe what the Heisenberg uncertainty relation asserts.

Exercise 19: Galilean relativity
A Galilean change of space-time coordinates ("Galilean boost") is given by

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{v}t\,, \quad t' = t \tag{1}$$

with a constant $\boldsymbol{v} \in \mathbb{R}^3$ called the relative velocity.

(a) Show that if V is translation invariant then Newton's equation of motion is invariant under Galilean boosts: If $t \mapsto (\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N)$ is a solution then so is $t \mapsto (\boldsymbol{Q}'_1, \ldots, \boldsymbol{Q}'_N)$.

(b) Show that if V is translation invariant and $\psi(t, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ is a solution of the Schrödinger equation, then so is

$$\psi'(t', \boldsymbol{x}'_1, \ldots, \boldsymbol{x}'_N) = \exp\left[\tfrac{i}{\hbar} \sum_{i=1}^{N} m_i(\boldsymbol{x}'_i \cdot \boldsymbol{v} - \tfrac{1}{2}\boldsymbol{v}^2 t')\right] \psi\Big(t', \boldsymbol{x}'_1 - \boldsymbol{v}t', \ldots, \boldsymbol{x}'_N - \boldsymbol{v}t'\Big). \tag{2}$$

(c) Show that if V is translation invariant and $t \mapsto Q(t) = (\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t))$ is a solution of Bohm's equation of motion for the wave function ψ, then $t \mapsto Q'(t) = (\boldsymbol{Q}'_1(t), \ldots, \boldsymbol{Q}'_N(t))$ is a solution of Bohm's equation of motion for ψ'. One says that Bohmian mechanics is Galilean invariant.

Exercise 20: Spinors
Verify that $|\boldsymbol{\omega}(\phi)| = \|\phi\|_S^2 = \phi^*\phi$. Proceed as follows: By (9.6), $\boldsymbol{\omega}(z\phi) = |z|^2\boldsymbol{\omega}(\phi)$, it suffices to show that unit spinors are associated with unit vectors. By (9.6) again, it suffices to consider ϕ with $\phi_1 \in \mathbb{R}$ (else replace ϕ by $e^{i\theta}\phi$ with appropriate θ). So we can assume, without loss of generality, $\phi = (\cos\alpha, e^{i\beta}\sin\alpha)$ with $\alpha, \beta \in \mathbb{R}$. Evaluate $\phi^*\boldsymbol{\sigma}\phi$ explicitly in terms of α and β, using the explicit formulas (9.3) for $\boldsymbol{\sigma}$. Then check that it is a unit vector.

Exercise 21: Can't Distinguish Non-Orthogonal State Vectors
(a) Alice gives to Bob a single particle whose spin state ψ is either $(1,0)$ or $(0,1)$ or $\frac{1}{\sqrt{2}}(1,1)$. Bob can carry out a quantum measurement of an arbitrary self-adjoint operator. Show that it is impossible for Bob to decide with certainty which of the three states ψ is.

(b) The same with only $(1,0)$ and $\frac{1}{\sqrt{2}}(1,1)$.

Hand in: Tuesday November 19, 2019, in class.

No reading assignment this week.