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Winter semester 2019/20
November 26, 2019

Foundations of Quantum Mechanics: Assignment 7

Exercise 26: Essay question. Describe the measurement problem of quantum mechanics. (Use formulas where appropriate.)

Exercise 27: Exponential distribution
Find the variance of the exponential distribution with parameter λ.

Exercise 28: GRW theory
Consider the GRW theory with the constant σ much smaller than the value 10^{-7} m suggested by GRW; say, $\sigma = 10^{-12}$ m. Explain why Heisenberg's uncertainty relation implies that a free electron, after being hit by a GRW collapse, could move very fast. Use the uncertainty relation to compute the order of magnitude of how fast it can be (assuming it was more or less at rest before the collapse); the mass of an electron is about 10^{-30} kg and $\hbar \approx 10^{-34}$ kg m^2 s^{-1}.

Exercise 29: Poisson process
For the Poisson process with rate $\lambda > 0$, determine for any fixed $t_0 > 0$ the distribution of $X_{t_0} = \#\{k : T_k < t_0\}$, the number of events up to time t_0. Follow two reasonings:

(a) Heuristically, assume that an event occurs in every infinitesimal time interval $[t, t + dt]$ independently of disjoint intervals with probability $\lambda\, dt$.
Hint: Divide $[0, t_0]$ in $n \gg 1$ subintervals of length $dt = t_0/n$.

(b) Rigorously, assume that the random variables $T_1, T_2, \ldots$ are defined to be $T_k = W_1 + \ldots + W_k$ with all waiting times W_k independent and exponentially distributed with parameter λ, i.e., with density $\rho(w) = 1_{w>0}\, \lambda\, e^{-\lambda w}$.
Hint:

$$\mathbb{P}\big(X_{t_0} \geq 2\big) = \mathbb{P}\Big(W_1 + W_2 < t_0\Big) = \int_0^{t_0} dw_1 \int_0^{t_0 - w_1} dw_2\, \rho(w_1)\, \rho(w_2) \quad \text{and}$$

$$\mathbb{P}(X_{t_0} = k) \;=\; \mathbb{P}\big(X_{t_0} \geq k\big) - \mathbb{P}\big(X_{t_0} \geq k+1\big)\,.$$

Hand in: Tuesday December 3, 2019, in class

Reading assignment due Friday December 6, 2019:

Sections 1–3 and 5 of J. Bell: Are There Quantum Jumps? In *Schrödinger. Centenary Celebration of a Polymath.* Cambridge University Press (1987). Reprinted as chapter 22 in J. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press (1987).