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Foundations of Quantum Mechanics: Assignment 10

Exercise 38: Essay question. Describe the Einstein–Podolsky–Rosen argument (either in terms of position and momentum or in terms of spin matrices).

Exercise 39: Symmetrizer and anti-symmetrizer
Let S_N be the group of permutations of $\{1,\ldots,N\}$. A function $\psi(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N)$ is called

$$\textit{symmetric} \text{ iff } \psi(\boldsymbol{x}_{\pi(1)},\ldots,\boldsymbol{x}_{\pi(N)}) = \psi(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N) \tag{1}$$

$$\textit{anti-symmetric} \text{ iff } \psi(\boldsymbol{x}_{\pi(1)},\ldots,\boldsymbol{x}_{\pi(N)}) = (-)^\pi \psi(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N)\,, \tag{2}$$

for every $\pi \in S_N$, where $(-)^\pi$ denotes the sign of π ($+1$ if π is even, -1 if odd). Let $\mathscr{S}_+$ denote the subspace of all symmetric functions in $L^2(\mathbb{R}^{3N})$ and $\mathscr{S}_-$ that of anti-symmetric functions. Show that the operators $P_\pm$ defined by

$$P_+\psi(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N) = \frac{1}{N!}\sum_{\pi\in S_N} \psi(\boldsymbol{x}_{\pi(1)},\ldots,\boldsymbol{x}_{\pi(N)}) \tag{3}$$

$$P_-\psi(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_N) = \frac{1}{N!}\sum_{\pi\in S_N} (-)^\pi\psi(\boldsymbol{x}_{\pi(1)},\ldots,\boldsymbol{x}_{\pi(N)}) \tag{4}$$

are the projections to $\mathscr{S}_\pm$. Use without proof that $(-)^\pi(-)^\rho = (-)^{\pi\circ\rho}$ for $\pi,\rho\in S_N$.

Exercise 40: Distinguish ensembles
A source generates

(a) either 10,000 particle pairs in the spin singlet state
$\frac{1}{\sqrt{2}}\big(|z\text{-up}\rangle|z\text{-down}\rangle - |z\text{-down}\rangle|z\text{-up}\rangle\big) = \frac{1}{\sqrt{2}}\big(|x\text{-up}\rangle|x\text{-down}\rangle - |x\text{-down}\rangle|x\text{-up}\rangle\big)$

(b) or randomly distributed 5,000 pairs in $|z\text{-up}\rangle|z\text{-down}\rangle$ and 5,000 in $|z\text{-down}\rangle|z\text{-up}\rangle$

(c) or randomly distributed 5,000 pairs in $|x\text{-up}\rangle|x\text{-down}\rangle$ and 5,000 in $|x\text{-down}\rangle|x\text{-up}\rangle$.

Alice and Bob are far from each other, and each receives one particle of every pair. By carrying out (local) Stern-Gerlach experiments on their particles and comparing their results afterwards, how can they decide whether the source was of type (a), (b), or (c)?

Exercise 41: Sum of projections
Let $\mathscr{H}$ be a Hilbert space of finite dimension, let P_1 and P_2 be projections in $\mathscr{H}$, $P_i = P_i^\dagger$ and $P_i^2 = P_i$, and let $\mathscr{H}_i$ be the range of P_i. Show that if $Q := P_1 + P_2$ is also a projection ($Q = Q^\dagger$ and $Q^2 = Q$), then **(a)** $\mathscr{H}_1 \perp \mathscr{H}_2$, and **(b)** the range $\mathscr{K}$ of Q is the span of $\mathscr{H}_1 \cup \mathscr{H}_2$.

Hand in: Tuesday January 7, 2020, in class

Reading assignment due Friday January 10, 2020:
N. Bohr: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review* **48**: 696–702 (1935)