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Foundations of Quantum Mechanics: Assignment 12

Exercise 47: Essay question. Describe Einstein's boxes argument.

Exercise 48: A measurement puzzle
Bob asks you to prepare an electron on which he will perform a single quantum measurement of either σ_x or σ_z without telling you which measurement he did. After his measurement, he will give you the electron back, so you can perform your measurement on it. Your task is to retrodict with certainty the result Bob got if he measured σ_x and the result he got if he measured σ_z. What should you do?

Exercise 49: No-cloning theorem
We show that it is impossible to duplicate the quantum state of an object without destroying the original quantum state. A *cloning mechanism* for the Hilbert space $\mathscr{H}_{\mathrm{obj}}$ would consist of a Hilbert space $\mathscr{H}_{\mathrm{app}}$, a ready state $\phi_0 \in \mathscr{H}_{\mathrm{app}}$ of the apparatus, a ready state $\psi_0 \in \mathscr{H}_{\mathrm{obj}}$ of the copy, and a unitary time evolution U on $\mathscr{H}_{\mathrm{obj}} \otimes \mathscr{H}_{\mathrm{obj}} \otimes \mathscr{H}_{\mathrm{app}}$ such that, for all $\psi \in \mathscr{H}_{\mathrm{obj}}$ with $\|\psi\| = 1$,

$$U(\psi \otimes \psi_0 \otimes \phi_0) = \psi \otimes \psi \otimes \phi_\psi \tag{1}$$

with some $\phi_\psi \in \mathscr{H}_{\mathrm{app}}$ that may depend on ψ. Prove that if $\dim \mathscr{H}_{\mathrm{obj}} \geq 2$, then no cloning mechanism exists. (*Hint*: Consider $\psi_1 \perp \psi_2$ and $\psi_3 = \frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$.)

Exercise 50: Main theorem about POVMs
The proof of the main theorem from Bohmian mechanics assumes that at the initial time t_i of the experiment, the joint wave function factorizes, $\Psi_{t_i} = \psi \otimes \phi$. What if factorization is not exactly satisfied, but only approximately? Then the probability distribution of the outcome Z is still approximately given by $\langle\psi|E(\cdot)|\psi\rangle$. To make this statement precise, suppose that

$$\Psi_{t_i} = c\psi \otimes \phi + \Delta\Psi\,, \tag{2}$$

where $\|\Delta\Psi\| \ll 1$, $\|\psi\| = \|\phi\| = 1$, and $c = \sqrt{1 - \|\Delta\Psi\|^2}$ (which is close to 1). Use the Cauchy-Schwarz inequality,

$$\big|\langle f|g\rangle\big| \leq \|f\|\,\|g\|\,, \tag{3}$$

to show that, for any $B \subseteq \mathscr{Z}$,

$$\Big|\mathbb{P}(Z \in B) - \langle\psi|E(B)|\psi\rangle\Big| < 3\|\Delta\Psi\|\,. \tag{4}$$

Hand in: Tuesday January 21, 2020, in class

No reading assignment this week