Prof. Dr. Roderich Tumulka
Eberhard Karls University Tübingen

Winter semester 2019/20
January 21, 2020

Foundations of Quantum Mechanics: Assignment 13

Exercise 51: Essay question. Explain why empirically indistinguishable ensembles pose a limitation to knowledge.

Exercise 52: 1D projection
Show that if $\|\psi\| = 1$, then $|\psi\rangle\langle\psi|$ is the projection to $\mathbb{C}\psi$.

Exercise 53: Tensor product
Suppose the Hilbert spaces $\mathscr{H}_a$ and $\mathscr{H}_b$ have finite dimensions d_a, d_b. Show that for any operators $T_a : \mathscr{H}_a \to \mathscr{H}_a$ and $T_b : \mathscr{H}_b \to \mathscr{H}_b$, there is a unique operator $T_a \otimes T_b : \mathscr{H}_a \otimes \mathscr{H}_b \to \mathscr{H}_a \otimes \mathscr{H}_b$ satisfying

$$(T_a \otimes T_b)(\psi_a \otimes \psi_b) = (T_a\psi_a) \otimes (T_b\psi_b) \tag{1}$$

for all $\psi_a \in \mathscr{H}_a$ and $\psi_b \in \mathscr{H}_b$, and that it has the following properties:

(i) $(T_a \otimes T_b)^\dagger = T_a^\dagger \otimes T_b^\dagger$

(ii) $(T_a \otimes T_b)(S_a \otimes S_b) = (T_aS_a) \otimes (T_bS_b)$

(iii) $\mathrm{tr}(T_a \otimes T_b) = (\mathrm{tr}\, T_a)(\mathrm{tr}\, T_b)$.

Hint: If $\{\phi_n^a : n = 1...d_a\}$ is an ONB of $\mathscr{H}_a$ and $\{\phi_m^b : m = 1...d_b\}$ one of $\mathscr{H}_b$, then $\{\phi_n^a \otimes \phi_m^b : n = 1...d_a, m = 1...d_b\}$ is an ONB of $\mathscr{H}_a \otimes \mathscr{H}_b$. You can express an operator T through its matrix $\langle\phi_n|T|\phi_{n'}\rangle$ relative to an ONB.

(*Remark:* For two non-interacting systems, the Hamiltonian is of the form $H = H_a \otimes I_b + I_a \otimes H_b$, and the time evolution is $e^{-iHt} = e^{-iH_at} \otimes e^{-iH_bt}$, i.e., of the form $U_t = U_{a,t} \otimes U_{b,t}$.)

Exercise 54: Reduced density matrix
Let $\psi \in \mathbb{S}(\mathscr{H}_a \otimes \mathscr{H}_b)$. Show that the reduced density matrix $\rho_\psi = \mathrm{tr}_b|\psi\rangle\langle\psi|$ is pure (i.e., a 1d projection) if and only if ψ factorizes, $\psi = \psi_a \otimes \psi_b$.

Hand in: Tuesday January 28, 2020, in class

Reading assignment due Friday January 31, 2020:
J. Bell: Against 'measurement.' *Physics World*, August 1990, pages 33–40.