Analysis 2

Kap. 0: Einleitung

$f: \mathbb{R}^n \to \mathbb{R}^m \qquad n, m \in \mathbb{R}$

$f: D \to \mathbb{R}^m \qquad D \subseteq \mathbb{R}^n$

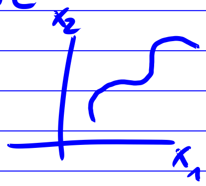
Bspe
- $m = 1$, $f(x_1 \dots x_n)$

  Skalarfeld $(x_1 \dots x_3)$
- Vektorfeld $\mathbb{R}^3 \to \mathbb{R}^3 \qquad \underline{E}(\underline{x})$

  $\underline{B}(\underline{x})$
- $\underline{x}(t) \in \mathbb{R}^3$, $\quad \underline{x}: \mathbb{R} \to \mathbb{R}^3$

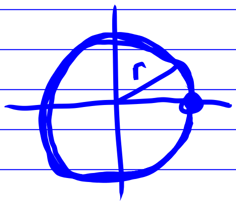
  parametrisierte Kurve

  $\{\underline{x}(t) \mid t \in \mathbb{R}\} \subset \mathbb{R}^3$

  Spur der Kurve

  $\underline{x}(t) = (r \cos \omega t, \; r \sin \omega t)$

  $\omega T = 2\pi$

  $\Rightarrow T = \frac{2\pi}{\omega}$

  ω "Kreisfrequenz"

• $\phi : \mathbb{R}^2 \to \mathbb{R}^3$ (parametrisierte) Fläche

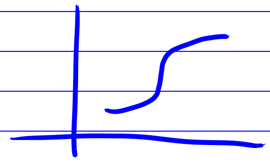
$\{ \phi(u,v) \mid (u,v) \in \mathbb{R}^2 \}$

• $T(x,y,z,t)$

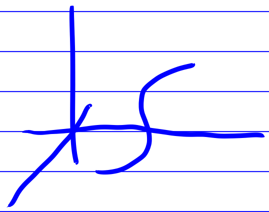
N Teilchen: $E : \mathbb{R}^{6N} \to \mathbb{R}$

<u>graphische Darst.</u>

• $\underline{x} : \mathbb{R} \to \mathbb{R}^2$

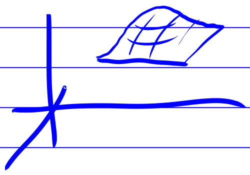


$\underline{x} : \mathbb{R} \to \mathbb{R}^3$



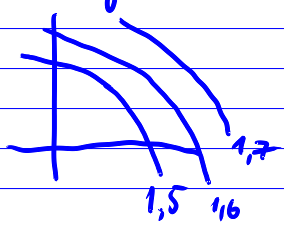
• $f : \mathbb{R}^2 \to \mathbb{R}$

Graph $\{ (x,y,f(x,y)) \mid (x,y) \in \mathbb{R}^2 \} \subset \mathbb{R}^3$

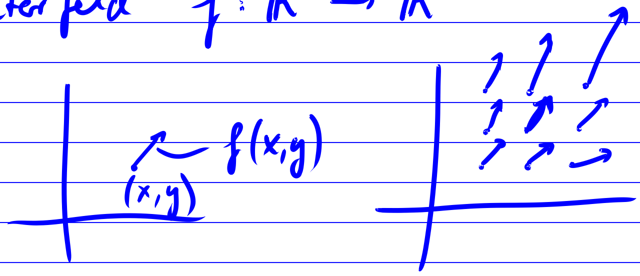


• Konturdiagramm $f : \mathbb{R}^2 \to \mathbb{R}$

$\{ (x,y) \mid f(x,y) = c \}$

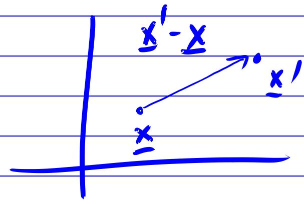


- Vektorfeld $f: \mathbb{R}^2 \to \mathbb{R}^2$

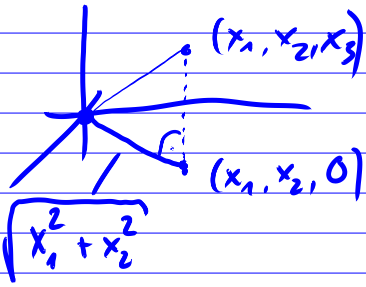


0.2 Abstand

in $\mathbb{R}^2$:

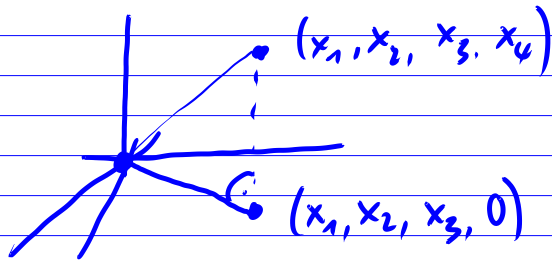


$$d(\underline{x}, \underline{x}') = \sqrt{(x_1' - x_1)^2 + (x_2' - x_2)^2}$$



$$d(\underline{x}, \underline{0}) = \sqrt{\left(\sqrt{x_1^2 + x_2^2}\right)^2 + (\ x_3\ )^2} = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$d(\underline{x}, \underline{x}') = \sqrt{\sum_{i=1}^{n} (x_i - x_i')^2}$$

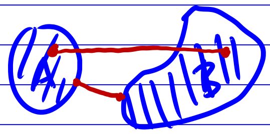


$$d(\underline{x}, \underline{x}') = \| \underline{x} - \underline{x}' \|$$

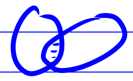
wobei $\|\underline{x}\| := \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$ Norm

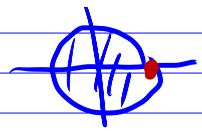
Abstand von Mengen $\underset{\neq \emptyset}{A}, \underset{\neq \emptyset}{B} \subseteq \mathbb{R}^n$

$$d(A,B) = \inf \{ d(x,x') \mid x \in A, x' \in B \}$$



$A \cap B \neq \emptyset \Rightarrow d(A,B) = 0$





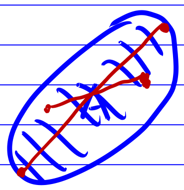
$A = \{ x \in \mathbb{R}^2 \mid \|x\| < 1 \}$

$B = \{ (1,0) \}$

$$d(\{x\}, \{x'\}) = d(x,x')$$

Durchmesser $\operatorname{diam}(A)$

$$= \sup \{ d(x,x') \mid x, x' \in A \}$$



Wenn $\{..\}$ nicht nach oben beschr., setze $\sup \{ \ldots \} = \infty$

$\Rightarrow$ $M \neq \emptyset$, $M \subseteq \mathbb{R}$ hat stets $\sup M \in \mathbb{R} \cup \{\infty\}$

Hier: $\forall \emptyset \neq A \subseteq \mathbb{R}^n$ hat stets ~~sup~~ $\operatorname{diam}(A) \in [0,\infty] := [0,\infty) \cup \{\infty\}$

Skalarprodukt oder Punktprodukt

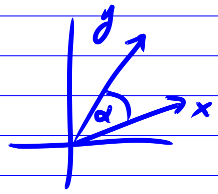
$$\underline{x} \cdot \underline{y} = \langle \underline{x}, \underline{y} \rangle = \sum_{i=1}^{n} x_i y_i$$

$$\underline{x} \cdot \underline{x} = \|\underline{x}\|^2$$

Geom. Bed.:

$$\underline{x} \cdot \underline{y} = \|\underline{x}\| \, \|\underline{y}\| \cos \alpha$$

$$\alpha = \sphericalangle(\underline{x}, \underline{y})$$



0.3 Arten von Fktnen

Bspe $\frac{x}{y}$, $e^{x^2 y}$

Polynom in $x_1 \dots x_n$

$$P(x_1 \dots x_n) = \sum_{\alpha_1 \dots \alpha_n = 0}^{N} c_{\alpha_1 \dots \alpha_n} \underbrace{x_1^{\alpha_1} \dots x_n^{\alpha_n}}_{\text{grad } \alpha_1 + \dots + \alpha_n}$$

Permutations-symmetrisch

$$f(x_{\sigma(1)}, \dots, x_{\sigma(n)}) = f(x_1 \dots x_n)$$

$\sigma \in S_n$

Bsp $\frac{\sin(x+y)}{\cos(xy)}$ ja, $\frac{x}{y}$ nein

Rotations-symmetrisch

$$f(R\underline{x}) = f(x_1 \dots x_n) \quad \forall R \in O(n)$$

$\Leftrightarrow \exists g: [0,\infty) \to \mathbb{R}$:

$$f(\underline{x}) = g(|\underline{x}|)$$

gerade und ungerade

$f: \mathbb{R}^n \to \mathbb{R}$ heißt gerade, wenn

$$f(-x) = f(x) \quad \forall x \in \mathbb{R}^n$$

ungerade, wenn $f(-x) = -f(x) \; \forall x \in \mathbb{R}^n$.

- $h, k : \mathbb{R} \to \mathbb{R}$

$$f(x,y) := h(x) + k(y)$$

oder $f(x,y) := h(x)\, k(y)$

$x, y \in \{1, \dots, N\}$

$f \in M(N, \mathbb{R})$, $\dim M(N, \mathbb{R}) = N^2$