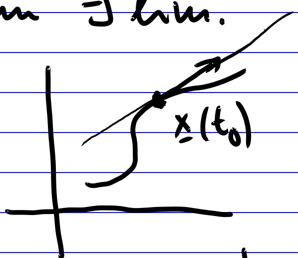
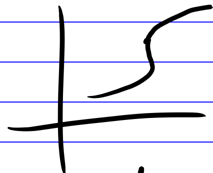
0.4 Ableitungen im $\mathbb{R}^n$

$\underline{x}(t)$, $\underline{x}: \mathbb{R} \to \mathbb{R}^n$

$$\frac{d\underline{x}}{dt}(t_0) := \lim_{h \to 0} \frac{\underline{x}(t_0+h) - \underline{x}(t_0)}{h}$$

wenn $\exists \lim$.





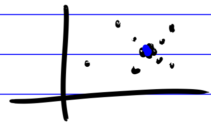
$$\text{Tangente} = \left\{ \underline{x}(t_0) + s\frac{d\underline{x}}{dt}(t_0) \,\middle|\, s \in \mathbb{R} \right\}$$

Vorl. Def. lim in $\mathbb{R}^n$:

$(\underline{x}_m)_{m \in \mathbb{N}}$ Folge in $\mathbb{R}^n$

$\underline{x}_m \xrightarrow{m \to \infty} \underline{x} \in \mathbb{R}^n$ oder $\underline{x} = \lim_{m \to \infty} \underline{x}_m$

$:\Leftrightarrow \quad x_{mi} \to x_i \quad \forall \quad i = 1..n$.



Konseq: $\frac{d\underline{x}}{dt}(t_0) = \left(\frac{dx_1}{dt}(t_0), \dots, \frac{dx_n}{dt}(t_0) \right)$

$\underline{x}(t) = (x_1(t), \dots, x_n(t))$

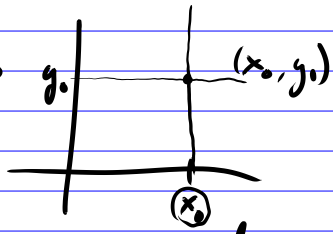
$\exists \frac{d\underline{x}}{dt}(t_0) \Leftrightarrow \forall i \in \{1..n\} \, \exists \frac{dx_i}{dt}(t_0)$.

· Andere Art von Abl.

Max $f(x,y)$. Ang. f ist bei $(x_0,y_0)=\underline{x}_0$ maximal.

Betrachte $g(x) := f(x, y_0)$

Also ist g maximal bei x_0



g diffbar $\Rightarrow \underbrace{g'(x_0)}_{\frac{\partial f}{\partial x}} = 0$

<u>Def</u> partielle Ableitung von f nach x:

$$\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$$

falls $\exists$ lim. Analog $\frac{\partial f}{\partial y}$.

$$\nabla f(x_0,y_0) = \left(\frac{\partial f}{\partial x}(x_0,y_0), \frac{\partial f}{\partial y}(x_0,y_0)\right)$$

heißt <u>Gradient</u> von f, ein Vektorfeld in $\mathbb{R}^2$.

$$\frac{\partial}{\partial x}\frac{x}{y} = \frac{1}{y}, \quad \frac{\partial}{\partial x} e^{x^2 y} =$$

$$e^{x^2 y}\, 2x\, y.$$

Also: an einem Max (x_0, y_0) von f (ang. $\exists\ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$) gilt:

$$\frac{\partial f}{\partial x}(x_0, y_0) = 0, \quad \frac{\partial f}{\partial y}(x_0, y_0) = 0$$

2. Abl.: $\quad \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x, y)$

$f(x_1 \dots x_n)$: $\quad \frac{\partial^2 f}{\partial x_i \partial x_j}$

$i = j$: $\quad \frac{\partial^2 f}{\partial x_i \partial x_i} = \frac{\partial^2 f}{\partial x_i^2}$

$$\frac{\partial}{\partial y}\frac{\partial}{\partial x} e^{x^2 y} = \frac{\partial}{\partial y}\left(e^{x^2 y}\, 2xy\right) =$$

$$= \left(e^{x^2 y}\ x^2\right) 2xy + e^{x^2 y}\left(2x\right)$$

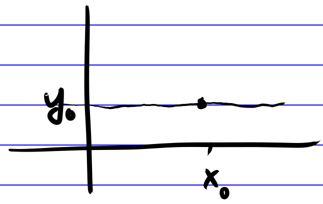
$$\frac{\partial^2}{\partial x^2} e^{x^2 y} = e^{x^2 y}(2xy)^2 + e^{x^2 y}\, 2y.$$

<u>Bem</u> ÜA: Für Poly $p(x_1 \dots x_n)$:

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = \frac{\partial^2 p}{\partial x_j \partial x_i}$$

<u>Richtungsableitung</u>

$\underline{x}(t) = (x_0 + t, y_0)$



$$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dt} f(\underline{x}(t))\Big|_{t=0}$$

<u>Def</u> Sei $0 \neq \underline{v} \in \mathbb{R}^2$. Die <u>Richtungsabl.</u> von f in Richtung $\underline{v}$ im Punkt $(x_0, y_0) = \underline{x}_0$ ist

$$\frac{\partial f}{\partial \underline{v}}(x_0, y_0) = \frac{d}{dt} f(\underline{x}_0 + t\underline{v})\Big|_{t=0}$$

ÜA: Für Poly $p(x,y)$ gilt $(\underline{v} = (v_x, v_y))$

$$\frac{\partial p}{\partial \underline{v}} = v_x \frac{\partial p}{\partial x} + v_y \frac{\partial p}{\partial y}.$$