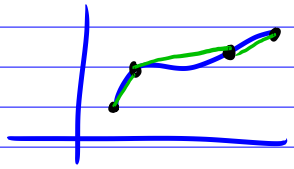
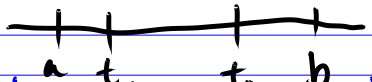
0.5 Kurvenlänge

$\underline{x} : [a,b] \to \mathbb{R}^n$

Länge L





$$U_N = \{ (t_0, t_1 \dots t_N) \in \mathbb{R}^{N+1} : a = t_0 < t_1 < t_2 < \dots < t_N = b \}, \quad U := \bigcup_{N=1}^{\infty} U_N$$

$$L \geq \sum_{i=1}^{N} d\left(\underline{x}(t_{i-1}), \underline{x}(t_i) \right)$$

Def $$L := \sup \left\{ \sum_{i=1}^{N} d(\underline{x}(t_{i-1}), \underline{x}(t_i)) \,\middle|\, (t_0, \dots, t_N) \in U \right\}$$

Satz Sei $\underline{x} : [a,b] \to \mathbb{R}^n$ derart, dass jede Komponentenfkt $x_i : [a,b] \to \mathbb{R}$ stetig diffbar ist. Dann ist

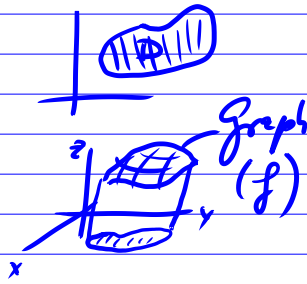
$$L = \int_a^b \left\| \frac{d\underline{x}}{dt} \right\| dt.$$

0.6 Flächenintegrale

Sei $D \subseteq \mathbb{R}^2$, $f : D \to \mathbb{R}$.

Flächenintegrale:

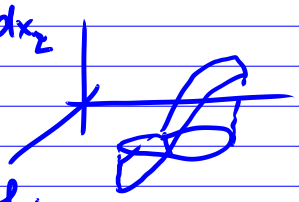
$$\int_D f(\underline{x})\, d\underline{x}$$



signierte Vol. $= d\underline{x}$

$= d^2\underline{x} = dA = dx_1 dx_2$

$= dx\, dy$



Mittelwert der Fkt f:

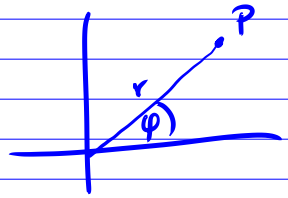
$$\overline{f} = \frac{1}{\text{area}(D)} \int_D f(\underline{x})\, d\underline{x}$$

0.7 Polarkoordinaten

V V.R., Basen, Bij. $V \to \mathbb{R}^n$

krummlinige Koordinaten

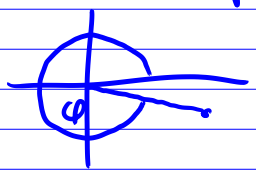
Bsp: Polarkoo. in $\mathbb{R}^2$.



r, φ

$$\boxed{\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}}$$

$r \in [0, \infty)$, $\varphi \in [0, 2\pi)$

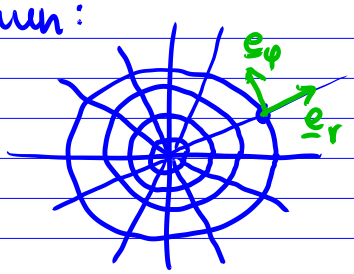


$$r = \sqrt{x^2 + y^2}$$

$$\varphi = \begin{cases} \arctan\frac{y}{x} & \text{falls } x, y > 0 \\ \pi/2 & \text{falls } x = 0, y > 0 \\ \pi + \arctan\frac{y}{x} & \text{falls } x < 0, \\ \frac{3}{2}\pi & \text{falls } x = 0, y < 0 \\ 2\pi + \arctan\frac{y}{x} & \text{falls } x > 0, y < 0 \end{cases}$$

Koordinatenlinien:

$$\underline{e}_r = \frac{\underline{x}}{\|\underline{x}\|},$$



$$\underline{e}_\varphi = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \underline{e}_r$$