Partielle und totale Ableitung

Def 4.31 Wenn $W = \mathbb{R}^m$, $G \subset \mathbb{R}^n$, so ist

$A = Df(x_0)$ eine $m \times n$ - Matrix

$$J(x_0) = \left(\frac{\partial f_i}{\partial x_j}(x_0) \right)_{ij}$$

genannt Jacobi-Matrix oder

Funktionalmatrix von f in x_0.

Satz 4.32 Sei $G \subset \mathbb{R}^n$ offen, $f: G \to \mathbb{R}$

part. diffbar. Wenn $\partial_i f$ st. in $x_0 \in G$

$\forall i \in \{1..n\}$, dann ist f tot. diffbar in x_0.

Bew: s. Skript.

Korollar 4.33 Sei $G \subset \mathbb{R}^n$ offen

$f: G \to \mathbb{R}^m$ st. part. diffbar, dann ist

f tot. diffbar in G, und insbes. st. in G.

Bew: s. Skript.

Also

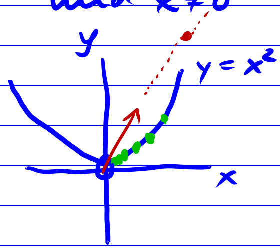
$$\begin{array}{ccccc} \text{st. part. diffb} & \Rightarrow & \text{total diffbar} & \Rightarrow & \text{part. diffb.} \\ & & \Downarrow & & \\ & & \text{st.} & & \end{array}$$

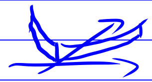
st. total diffbar $\Longleftrightarrow$ st. part. diffb.

("st. diffbar") $\quad C^1(G, W)$

Bsp part. diffbar $\not\Rightarrow$ st.

$$f(x,y) = \begin{cases} 1 & \text{falls } y = x^2 \text{ und } x \neq 0 \\ 0 & \text{sonst.} \end{cases}$$





nicht st. in $(0,0)$ $\quad \left(f\left(\frac{1}{n}, \frac{1}{n^2}\right) \longrightarrow 1 \neq 0 = f(0,0) \right)$

$\exists \ \frac{\partial f}{\partial v}(0) = 0 \ \forall v \in \mathbb{R}^2$.

aber $\nexists \ Df(0)$